Incremental Cooperative Relaying in Time-Correlated Rayleigh Fading Channels

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***Abstract*—Incremental relaying is a communication technique enabling cooperative diversity in wireless networks. The relay forwards a correctly decoded packet only after getting the feedback that the destination failed to receive the data directly from the source. We model an incremental relaying protocol as a time-discrete finite-state Markov process and analyze its throughput performance in time-correlated radio channels with Rayleigh fading. We also determine two boundary cases: fully correlated and fully uncorrelated fading channel. The analysis shows that the throughput gains of incremental relaying to conventional Automatic Repeat-reQuest retransmission strongly depend on the channel correlation, relay position, and source-to-destination fading margin.**

***Index Terms*—Cooperative diversity, incremental relaying, Rayleigh fading, Markov process, ARQ, antenna diversity.**

I. Introduction

Cooperative diversity is getting significant attention as a promising technique for multi-path fading mitigation in wireless networks. In the wireless medium, nodes can overhear transmissions between a communicating pair of nodes and forward the received data to the destination. Thus, the destination can obtain copies of the data via uncorrelated paths and benefit from additional signal diversity. Cooperative diversity is especially attractive in wireless networks with low-cost devices, where other fading mitigation techniques (e.g., equalization, MIMO) cannot be applied due to strict constraints on hardware size and complexity (see [1], [2]).

In this paper, we study an incremental relaying (IR) protocol, where the relay forwards a correctly decoded copy of a packet only after the direct packet transmission from the source to the destination failed [3]. In particular, we investigate the throughput of IR in a time-correlated Rayleigh fading channel modeled as a two-state Markov process (see [4], [5]). We model incremental relaying as a finite-state Markov process incorporating three channels with Markovian errors. In addition, two boundary cases are analyzed: fully correlated and uncorrelated fading channels. The resulting performance of IR is compared to that of Stop-and-Wait Automatic Repeat-reQuest (SW ARQ) and simple receiver antenna diversity.

The rest of the paper is organized as follows. Section II summarizes related work. Section III explains the channel model and protocol operation. Section IV gives the analytical framework for the incremental relaying protocol. Section V examines the performance of incremental cooperative relaying. Section VI concludes the paper.

II. Related Work

Although there are numerous publications that propose and analyze cooperative relaying protocols, only few of them consider the time-correlation property of wireless channels.

Zimmermann et al. [6] propose a coded cooperative Hybrid-ARQ scheme with power control and compare it via simulations with conventional Hybrid-ARQ in various scenarios. The authors model a randomly distributed channel coherence time to reflect the channel correlation. Dianti et al. [7] use Markov chains to analyze a node-cooperative stop-and-wait (NCSW) scheme, where the source and multiple relays utilize distributed space-time codes and transmit to the destination simultaneously. The implementation of such a mechanism requires significant coordination, complex receivers, and synchronization among transmitting nodes. Finally, Mahinthan et al. [8] analyze quadrature signaling (QS)-based cooperative ARQ in time-correlated Nakagami-m fading channels. For a particular mobility scenario, the authors compare via simulations the performance of their protocol with NCSW [7] and IR. The results show that with a properly selected relay the QS-based scheme performs better in terms of packet loss, delay, and jitter. The drawback of QS-based cooperative diversity is its limitation to QPSK and strict synchronization requirements.

III. Modeling Assumptions

A. Channel Model

We consider wireless channels with Rayleigh block fading. Time is divided into discrete steps indexed by $k \in \mathbb{N}$. The channel state $\mathcal{C}(k)$ at time instant k can be either "good" or "bad". The transmission between a source and a destination is modeled as a series of signal-to-noise ratio samples $\{\mathrm{SNR}(k)\}$, each of certain duration T for which the signal level remains constant. A binary random process $\{\mathcal{C}(k)\}$ is defined as

$$\mathcal{C}(k) = \begin{cases} \text{“Good”} & \text{if } \mathrm{SNR}(k) \geq \mathrm{SNR}_{\min}, \\ \text{“Bad”} & \text{if } \mathrm{SNR}_k < \mathrm{SNR}_{\min}, \end{cases} \quad (1)$$

where $\mathrm{SNR}_{\min}$ is the receiver SNR threshold. Whenever the channel is in the good state, a packet is received without error. Whenever the channel is in the bad state, an outage event takes place, and the transmitted packet cannot be received correctly.

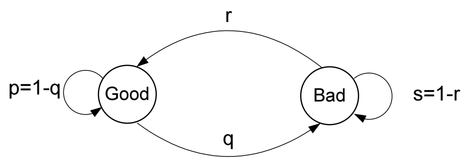

Fig. 1. Model of an erroneous channel as a two-state Markov process.

The time-correlation property of the process $\{\mathcal{C}(k)\}$ is modeled by a two-state Markov process shown in Figure 1 (also see [4], [5]). We assume that a time slot T equals the transmission time of one data frame. The corresponding transition probability matrix

$$\mathbf{C} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 1-q & q \\ r & 1-r \end{bmatrix} \quad (2)$$

contains the state transition probabilities as defined in Fig. 1.

The outage probability for a radio channel with Rayleigh fading is

$$\epsilon = 1 - \exp\left(-\frac{1}{\psi}\right), \quad (3)$$

where the term ψ is called fading margin and is given by

$$\psi = \frac{E\left[\mathrm{SNR_{SD}}\right]}{\mathrm{SNR_{min}}}. \quad (4)$$

The term $E[\mathrm{SNR_{SD}}]$ denotes the expected SNR at the destination and is calculated according to the pathloss by

$$E\left[\mathrm{SNR_{SD}}\right] = \mathrm{SNR_S}\left(\frac{d_{\mathrm{SD}}}{d_0}\right)^{-\alpha}, \quad (5)$$

where $\mathrm{SNR_S}$ is the SNR at the sender, d_{SD} is the distance between sender and destination, $d_0 = 1\,\mathrm{m}$ is a reference distance, and α is the pathloss exponent.

The paper [4] shows how to derive the transition matrix $\mathbf{C}$ for given fading margin ψ, Doppler spread f_m, and frame duration T. In this paper we apply this approach. As in [4], the fading is considered as slow fading if $f_m T < 0.1$ and as fast fading if $f_m T > 0.2$.

B. Incremental Relaying

We study decode-and-forward cooperative communication with feedback and one relay as illustrated in Figure 2. The wireless channels between the nodes experience Rayleigh fading and are characterized by outage probabilities ϵ_P, ϵ_I, and ϵ_R for the primary, interim, and relay channel, respectively. Each channel is modeled as a two-state Markov process with transition matrix $\mathbf{C}_P$, $\mathbf{C}_I$, and $\mathbf{C}_R$, respectively.

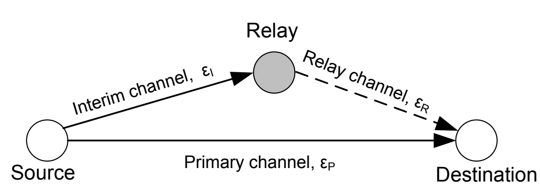

Fig. 2. Cooperative diversity channel with one cooperative relay.

Assumptions on the cooperative relaying operation are:

- Data transmissions are strictly orthogonal in time.
- Incorrectly received packets are dropped.
- Source and relay use the same transmission rate and power.
- The relay selection procedure is not modeled. The cooperative relay is "given" *a priori*.
- Feedback channels use a stronger encoding schemes and assumed to be error-free.
- Feedback duration is either negligible or included in the packet transmission time T.
- All three channels experience the same value of $f_m T$.
- At any time point, there is at least one data packet queued for transmission by the source.

The first two points are characteristic for low-cost radios [2].

The incremental relaying protocol operates as follows:

1) The source broadcasts a new data packet to relay and destination and waits for their feedback.
2) If the relay receives the packet correctly, it stores it in a buffer and notifies the source.
3) If the destination receives the packet correctly, it notifies the source and the relay, and a new transmission begins.
4) If the destination does not receive the message, but the relay does, the relay forwards the stored packet and drops it from the buffer.
5) If neither source nor destination receive the packet correctly, the source retransmits the failed packet. There is no limitation on the number of retransmission attempts.

Note that the relay drops the packet after forwarding it. This assumption eases the mathematical analysis. In practice, it can be more beneficial if a relay does not empty its buffer and keeps retransmitting until the destination receives the packet correctly. This variation can be analyzed in a similar manner as the proposed protocol.

IV. Analytical Framework

In this section, first, a framework for IR analysis as a Markov process is given. In Subsection IV-B, two boundary scenarios for fading channel correlation are introduced. Finally, throughput expressions for SW ARQ and receiver antenna diversity are given in Subsection IV-C and IV-D, respectively.

A. Incremental Relaying as a Markov Process

At any time slot k, the incremental relaying protocol operates in one of the following states $\mathcal{P}(k)$:

- Tx: transmission of a new packet from the source to the destination.
- R: relaying of the packet by the relay to the destination.
- RT: retransmission of the packet from the source.

Each state transition takes one time slot.

Table I contains the protocol transition rules for all possible combinations of the relaying protocol and radio channel states. It incorporates the protocol operation on three radio channels

TABLE I

STATE TRANSITIONS IN INCREMENTAL RELAYING. THE TRANSITIONS DEPEND ON THE STATE OF THE PROTOCOL (TX = TRANSMIT, R = RELAY, RT = RETRANSMIT) AND THE STATES OF THE PRIMARY, INTERIM, AND RELAY CHANNELS (G = GOOD CHANNEL, B = BAD CHANNEL)

state	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
$\mathcal{P}(k)$	Tx	Tx	Tx	Tx	Tx	Tx	Tx	Tx	R	R	R	R	R	R	R	R	RT	RT	RT	RT	RT	RT	RT	RT
$\mathcal{C}_P(k)$	G	G	G	G	B	B	B	B	G	G	G	G	B	B	B	B	G	G	G	G	B	B	B	B
$\mathcal{C}_I(k)$	G	G	B	B	G	G	B	B	G	G	B	B	G	G	B	B	G	G	B	B	G	G	B	B
$\mathcal{C}_R(k)$	G	B	G	B	G	B	G	B	G	B	G	B	G	B	G	B	G	B	G	B	G	B	G	B
$\mathcal{P}(k+1)$	Tx	Tx	Tx	Tx	R	R	RT	RT	Tx	RT	Tx	RT	Tx	RT	Tx	RT	Tx	Tx	Tx	Tx	R	R	RT	RT
reward	1	1	1	1	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	1	0	0	0	0

and defines a new 24-state Markov process. Based on the table, the elements of the corresponding transition probability matrix

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,24} \\ \vdots & \vdots & \ddots & \vdots \\ a_{24,1} & a_{24,2} & \cdots & a_{24,24} \end{bmatrix}. \tag{6}$$

are derived by

$$a_{i,j} = \begin{cases} 0 \quad \text{if } \mathcal{P}_i(k+1) \neq \mathcal{P}_j(k), \\ \mathrm{P}\big[\mathcal{C}_P^{i,j}\big] \cdot \mathrm{P}\big[\mathcal{C}_I^{i,j}\big] \cdot \mathrm{P}\big[\mathcal{C}_R^{i,j}\big] \quad \text{otherwise,} \end{cases} \tag{7}$$

where $\mathrm{P}\big[\mathcal{C}_P^{i,j}\big]$, $\mathrm{P}\big[\mathcal{C}_I^{i,j}\big]$, and $\mathrm{P}\big[\mathcal{C}_R^{i,j}\big]$ denote the transition probability from state i to state j $(i, j \in \{1, \ldots, 24\})$ for the primary, interim, and relay channel, respectively, and are obtained from the corresponding channel transition matrices $\mathbf{C}_P$, $\mathbf{C}_I$, and $\mathbf{C}_R$.

The Markov process in Table I is irreducible and aperiodic, and its limiting-state probabilities $\mathbf{b} = [b_1\ b_2\ b_3\ \ldots\ b_{24}]$ can be obtained by solving the set of linear equations

$$\mathbf{bA} = \mathbf{b}, \\ \text{with } \sum_{i=1}^{24} b_i = 1. \tag{8}$$

Whenever a packet is successfully delivered to the destination, the protocol returns to state Tx. We assign a reward r_i (packet delivery to the destination) obtained by the system in state i as

$$r_i = \begin{cases} 1 \quad \text{if } \mathcal{P}_i(k+1) = \text{Tx}, \\ 0 \quad \text{otherwise.} \end{cases} \tag{9}$$

Since the time spent in the state before a transition (holding time) is the same for each transition and equals one frame slot, the average reward over one time slot is

$$\eta_{IR} = \sum_{i=1}^{24} r_i b_i, \tag{10}$$

which corresponds to the expected throughput (delivered packets per time slot) of the IR protocol. More information on the use of renewal-reward theory can be found in [9] and [10].

B. Boundary Scenarios

1) Fully Correlated Channel: We define the fully correlated radio channel as a channel which always remains in one state, be it good or bad. This can happen if nodes' positions and the surrounding environment remain static. The state of each radio channel in this scenario is defined in the first time slot by the respective channel outage probability. The expected throughput η_{IR} is obtained by

$$\eta_{IR} = 1 - \epsilon_P + \frac{1}{2}\epsilon_P\,(1-\epsilon_I)\,(1-\epsilon_R)\,. \tag{11}$$

When the relay is used, two time slots are needed for an end-to-end transmission. Therefore, the respective part in (11) is scaled by $1/2$.

2) Independent and Identically Distributed (i.i.d.) Channel: A radio channel is i.i.d. when its channel states are independent of each other and have same probability distribution, which, in our case, is the channel outage probability. It is a boundary case for fast fading (on the frame level) with an uncorrelated (memoryless) channel. We assume that the channel always remains constant for the duration of at least one frame slot.

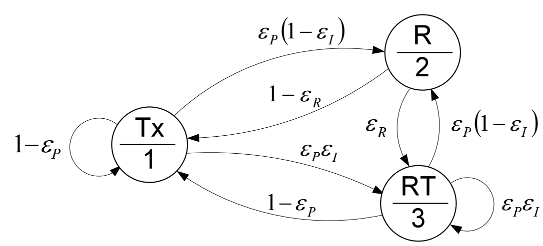

Fig. 3. Incremental relaying protocol as a three-state Markov chain.

Figure 3 represents the incremental relaying as a three-state Markov chain. The corresponding transition probability matrix for i.i.d. radio channels is

$$\mathbf{P} = \begin{bmatrix} 1-\epsilon_P & \epsilon_P\,(1-\epsilon_I) & \epsilon_P\epsilon_I \\ 1-\epsilon_R & 0 & \epsilon_R \\ 1-\epsilon_P & \epsilon_P\,(1-\epsilon_I) & \epsilon_P\epsilon_I \end{bmatrix}. \tag{12}$$

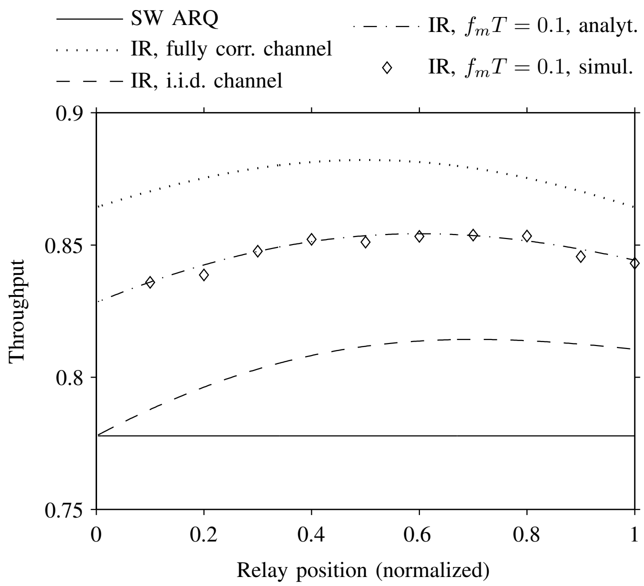

Fig. 4. Throughput vs. normalized relay position; $\psi = 6\,\text{dB}$; $\alpha = 3$.

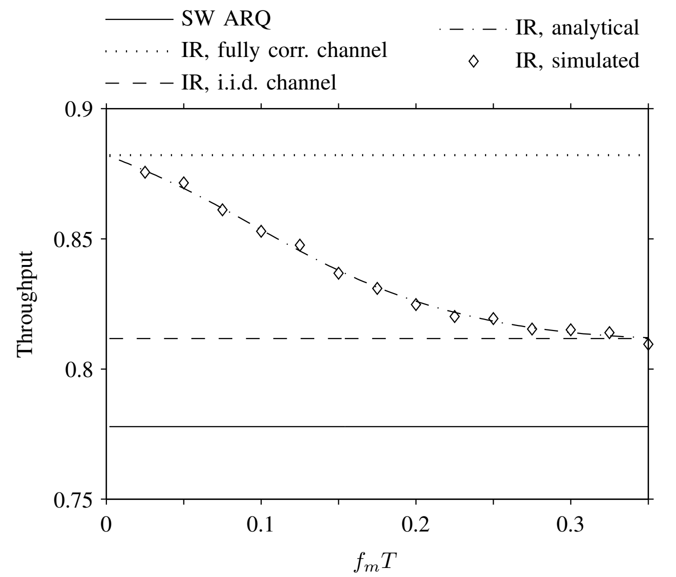

Fig. 6. Throughput vs. fading dynamics f_mT; $d_{\text{SR}} = 0.5 d_{\text{SD}}$; $\psi = 6\,\text{dB}$; $\alpha = 3$.

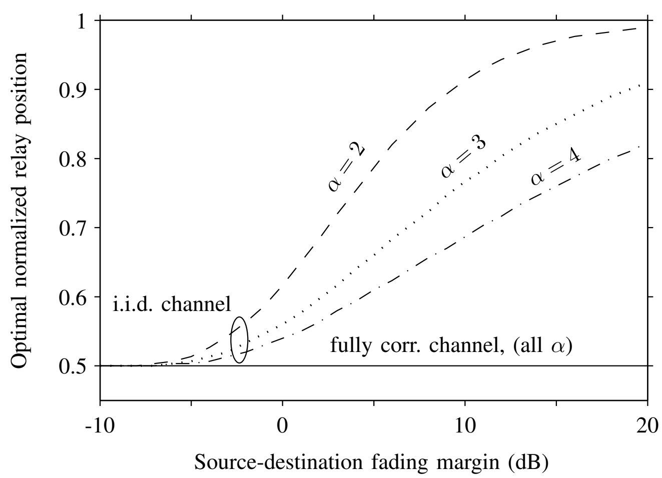

Fig. 5. Optimal normalized relay positions for boundary fading scenarios.

The limiting-state probability for state Tx is the probability of a new packet transmission and, therefore, is also the throughput of the protocol:

$$\eta_{IR} = \frac{1 + \epsilon_P \epsilon_I \epsilon_R - \epsilon_P \epsilon_I - \epsilon_P \epsilon_R}{1 + \epsilon_P - \epsilon_P \epsilon_I}. \tag{13}$$

C. Time Diversity via ARQ

ARQ and Hybrid-ARQ are well-accepted error control protocols that exploit time diversity. There are three basic ARQ protocols: stop-and-wait (SW), selective-repeat (SR), and go-back-N (GBN). The performance of these protocols in wireless channels with Markovian errors has been extensively evaluated analytically and via simulations (see, e.g., [4], [11]).

The SR scheme provides the best throughput at the receiver, but leads to longer delays. We assume the feedback channel is error-free, and the feedback duration is neglected or included into the data frame transmission. Under such assumptions, the resulting throughput η of SW and SR schemes is the same [7], and is simply obtained by

$$\eta = 1 - \epsilon. \tag{14}$$

We use this throughput as a comparison baseline to evaluate the performance of incremental relaying.

D. Receiver Antenna Diversity

In receiver antenna diversity, the receiving node is equipped with several antennas and receives transmitted signals on them simultaneously, or is able to choose the antenna with the strongest signal based on received preambles. To make a fair comparison to cooperative relaying, we assume that the receiver obtains independent copies of the signal on two antennas and applies the selection combining on a frame level. This means a frame is received erroneously only when the SNR on both antennas is lower than the required threshold. In this case the frame is dropped and a retransmission from the source starts again.

Although more sophisticated multiple-antenna techniques exists, the advantage of receiver antenna diversity is easy implementation and use in cheap receivers. However, to obtain independent signals a minimum distance between antennas is required, which brings an additional constraint on hardware size.

The resulting throughput of receiver antenna diversity is

$$\eta_{RxD} = 1 - \epsilon_P^2 \tag{15}$$

and is independent of the fading dynamics on the channel.

V. Performance Results

Figure 4 shows the impact of relay location on throughput. The relay is located on the line between the source and destination at the normalized distance $d_{\text{SR}}/d_{\text{SD}}$. For a given relay position and source-destination fading margin, the throughput of IR in time-correlated Rayleigh channels is upper and lower bounded by the performance in fully-correlated and i.i.d. fading channels, respectively. If the relay-to-destination distance is larger than the source-to-destination distance ($\epsilon_R > \epsilon_P$), incremental relaying in an i.i.d. channel performs worse than simple SW ARQ. Figure 4 also shows the IR throughput for a moderately correlated channel with $f_mT = 0.1$. The simulation and analytical results closely match each other and have a similar trend as the bounds.

Numerical analysis shows that the maximum throughput of the fully-correlated-channel scenario is achieved when the relay is located in the middle between source and destination. In contrast, for i.i.d. channels, the optimal relay placement depends on the fading margin and the pathloss exponent, as it is shown in Figure 5. Evidently, for intermediate channels, optimal relay positions lie between those of fully correlated and i.i.d. channels. A formal proof for the relay location maximizing the throughput is out of the scope of this paper.

Figure 6 shows throughput for $\psi = 6\,\text{dB}$ versus f_mT. The figure clearly indicates the transition from fully correlated channel to the i.i.d. channel bounds with growing value of f_mT, i.e., increasing fading dynamics.

Figure 7 compares throughput bounds of IR, SW ARQ, and receiver antenna diversity versus fading margin ψ. The IR performance in fully correlated channels serves as an upper bound for $\psi \gtrapprox -2\,\text{dB}$. In that region, IR performs better in highly correlated slow fading channels, since it provides path diversity via the relay when the direct channel is bad for long time. However, at some point, the bounds switch, and the throughput of the i.i.d. channel becomes the upper bound. Here, cooperative relaying works better in less correlated channels. The crossing occurs when the number of frames per time delivered via the relay is the same as the respective value for frames delivered to the destination directly by the source. Below this point ($\psi \lessapprox -2\,\text{dB}$) the two-hop communication starts to dominate the efficiency of the end-to-end communication. The direct communication vanishes with $\psi \lessapprox -7.5\,\text{dB}$.

Finally, we see that the receiver antenna diversity significantly outperforms IR for $\psi > 0\,\text{dB}$. This is because the receiver obtains two copies of the same data without a delay. At lower values, IR operates better since the two-hop gain takes the dominating role.

VI. Conclusions

In this paper we analyzed the performance of an incremental relaying protocol in time-correlated Rayleigh fading channels. We introduced an analytical model of the protocol as a finite-state Markov process and used this model to obtain the resulting throughput in different fading conditions. We also introduced two boundary cases for the channel time correlation: fully correlated and i.i.d. fading channels. The results show that the throughput of incremental relaying significantly varies depending on channel fading dynamics. With a properly located relay, incremental relaying always outperforms conventional ARQ schemes. But simple receiver antenna diversity still can perform significantly better than

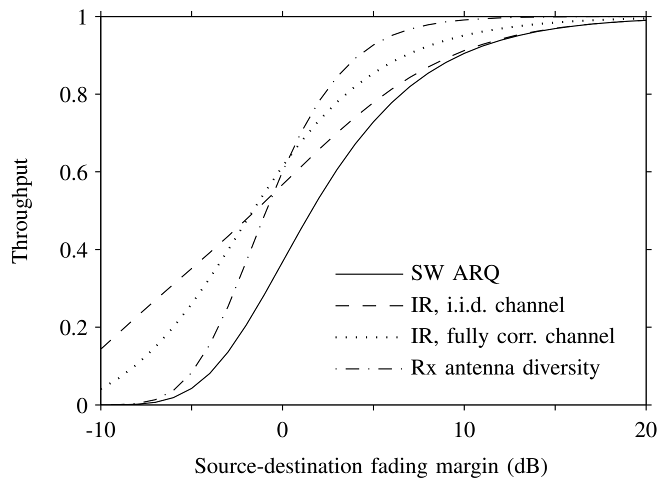

Fig. 7. Throughput of incremental relaying, SW ARQ and receiver antenna diversity; $d_{\mathrm{SR}} = 0.5 d_{\mathrm{SD}}$; $\alpha = 3$.

cooperative schemes, and can be used in nodes where the hardware constraints allow it.

A relay selection process based on instantaneous channel information can further improve the throughput of IR, and will be considered in further research.

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