

# Modeling Self-Organizing Systems

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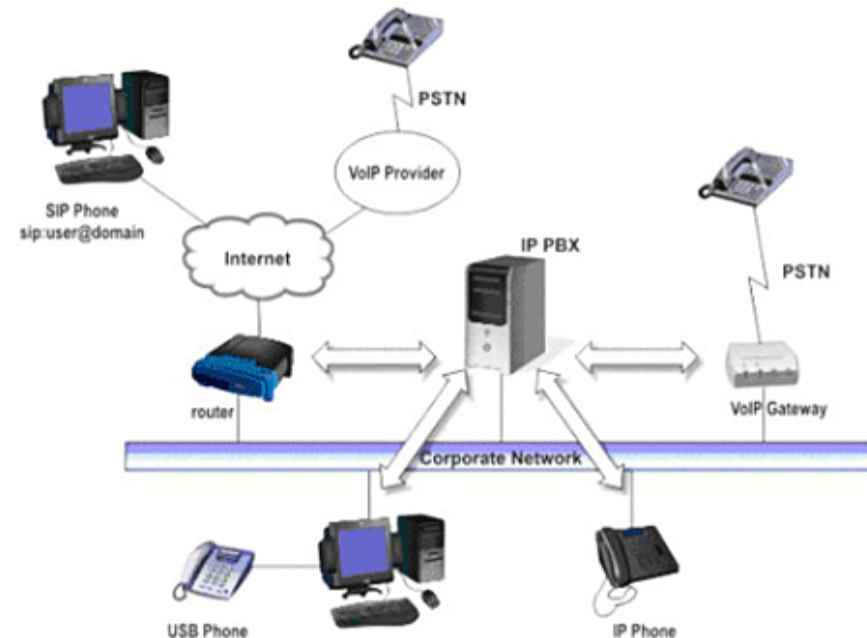


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# What is a complex system?

- A **complex system** consists of a large number of interacting entities.
- **Local rules**, which describe the behavior of each entity in the system, lead to complex global states.
- A self-organizing system exhibits **emergence**: A global pattern in the collective behavior, which results from the local interactions.
- The emergent behavior does not result from the existence of a central controller.



# Self-Organizing Systems

## Autonomy:

No external entities control the behavior of the system

## Adaptivity:

System keeps working after changes in the environment

## Self-Organization

## Emergence:

Local interactions induce the creation of globally coherent patterns

## Decentralization:

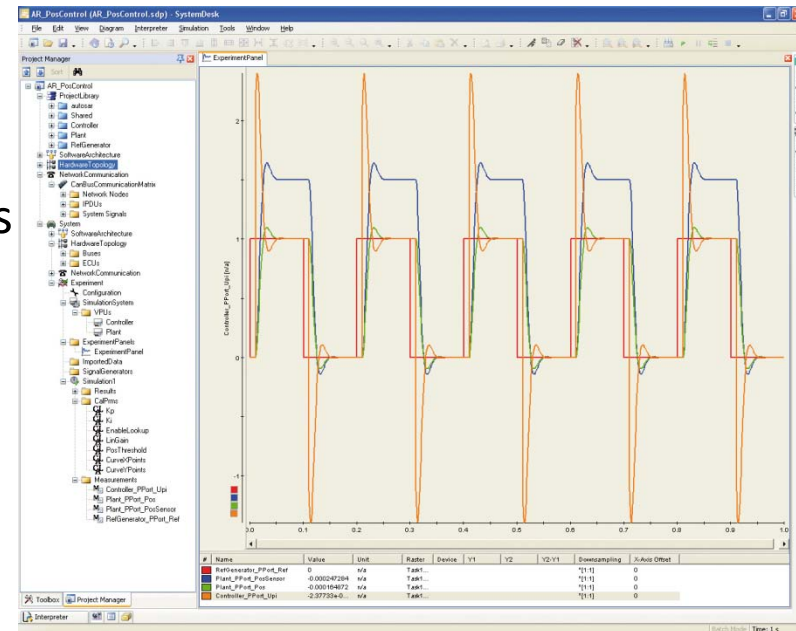
No central entity controls the behaviour of the system



# Difference between model and simulation

For a **simulation**,

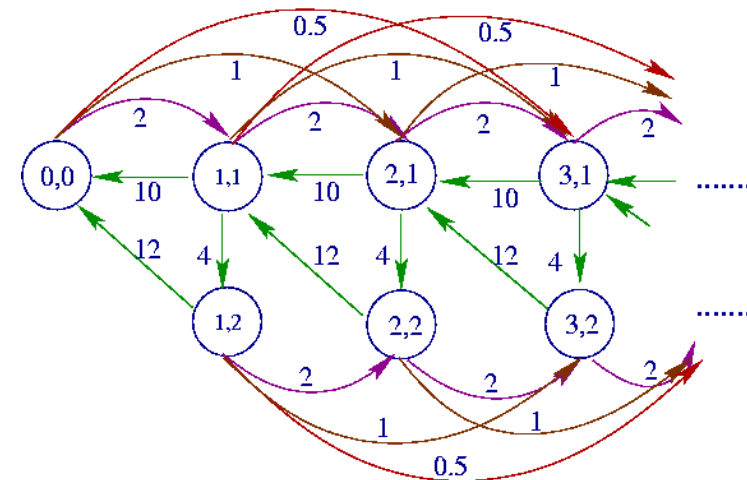
- the **behavior** of each entity must be described,
- the algorithms of the behaviors of all entities in the system are **implemented** as good as possible (with respect to the relevant parameters/properties),
- system properties can be analyzed by different „**simulation runs**“,
- the **derived properties** correspond to the individual cases of the simulation runs.



# Difference between model and simulation

A formal **model** is a representation of the system, which

- **simplifies** the system on the relevant properties and parameters,
- is **mathematically formalized**,
- allows the mathematical derivation of **global properties** from the local rules.



# Discrete or Continuous?

## Macro level modeling

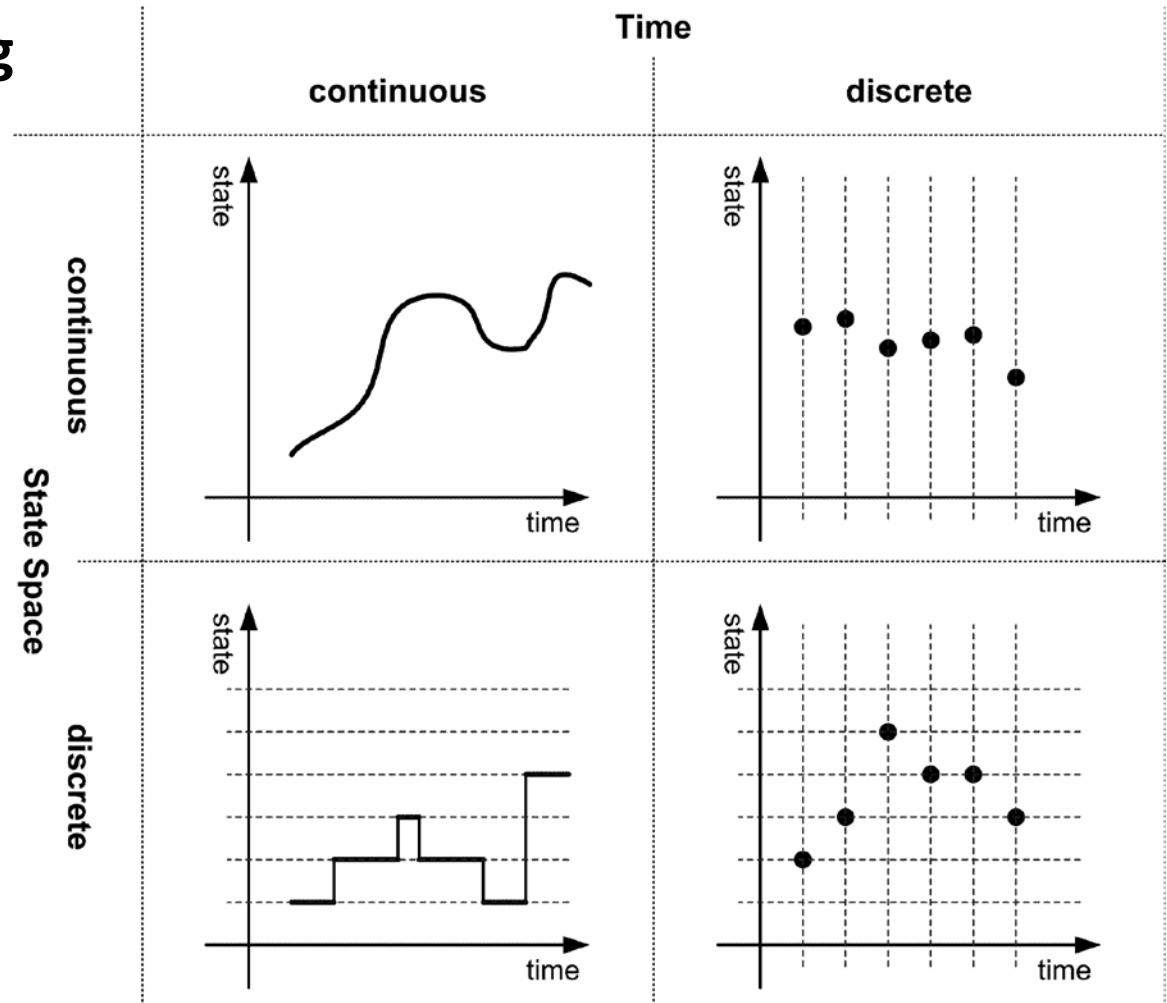
- Discrete/continuous time
- Discrete/continuous state space

	Discrete	Continuous
Time	Time increases in discrete steps. This could also be event based: Each step corresponds to one event.	Continuous change of the system can happen everytime
States	At each point of time there are only finite or countable many possibilities for the current state.	Uncountable state space.



# Discrete or Continuous?

## Macro level modeling





## Micro level modeling

	Discrete	Continuous
<b>Time</b>	Time increases in discrete steps. This could also be event based: Each step corresponds to one event.	Continuous change of the system can happen everytime
<b>Object set</b>	Only a finite number of objects are modeled	Uncountable number of objects
<b>States</b>	Finite memory: Each object is in one of a finite number of states	Uncountable memory for each object
<b>Interaction</b>	Each object can interact with a finite number of other objects	Uncountable many interactions

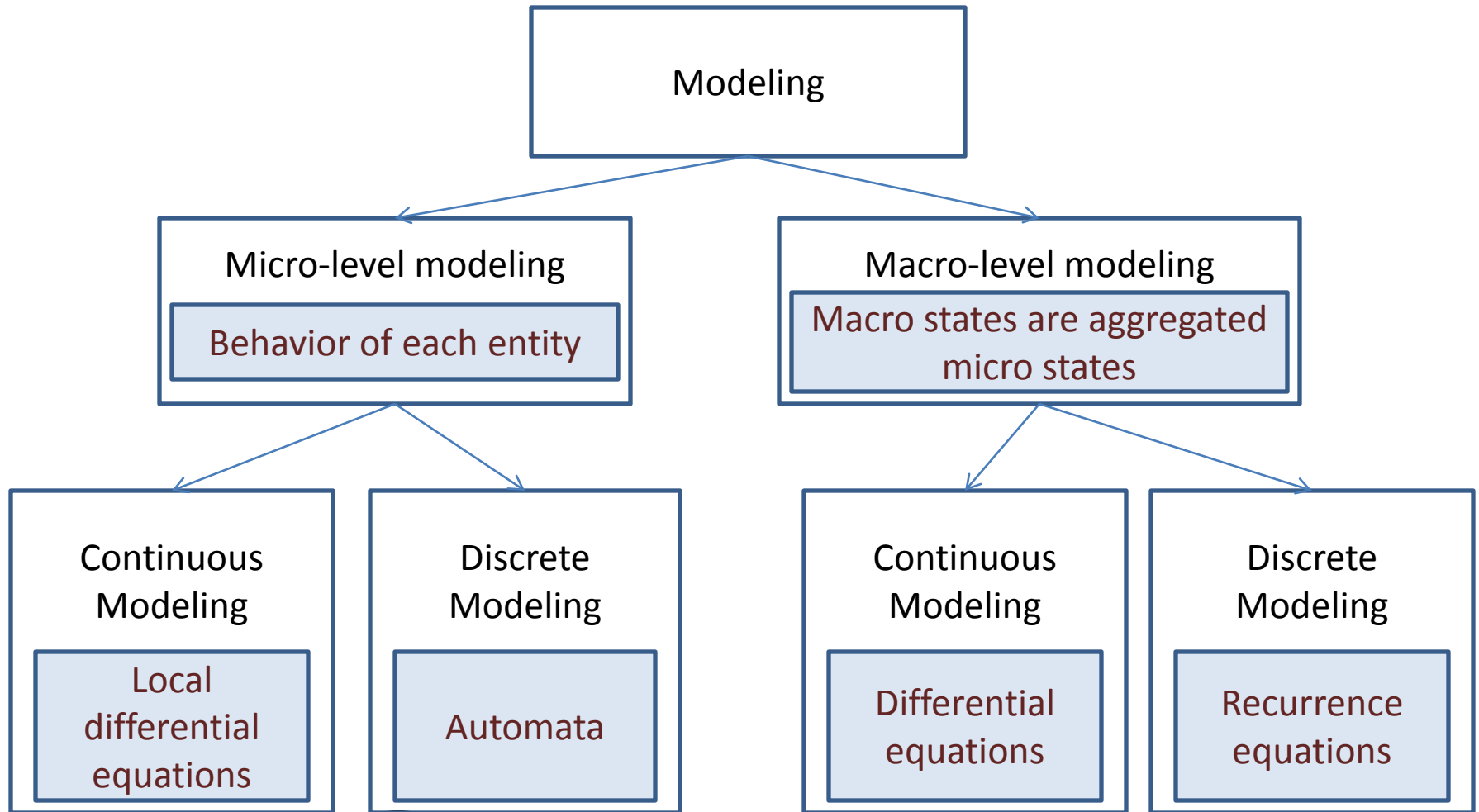
Questions before the design of the model:

- Which system do I want to model?
- Which properties should be investigated?

⇒ Decision, whether **discrete** or **continuous** modeling fits better



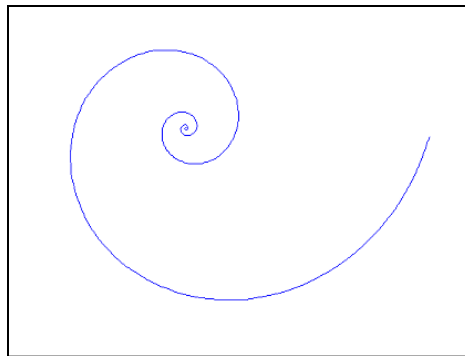
# Modeling approaches



A **dynamical system** consists of

- a state space  $S$ ,
  - The state space contains all possible states of the system.
  - If more than one value is needed to describe the state, then it can be of higher dimension, e.g., each state  $s = (s_1, \dots, s_n) \in S$  is a tuple.
- a time variable  $t$ ,
- an evolution law, which describes the change of the state over time.

The orbit of the system is the path of the state over time.



In a **continuous system**,

- the time variable is a real number,
- the evolution rule is a differential equation:

$$\frac{dx}{dt} = \dot{x} = X(x)$$

In a **discrete system**

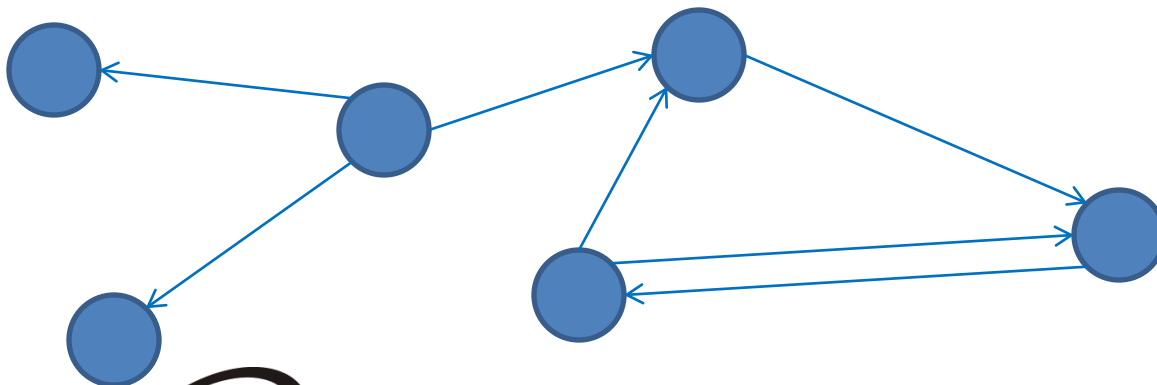
- the time variable is an integer,
- the evolution rule is a recurrence equation:

$$x_{t+1} = f(x_t)$$



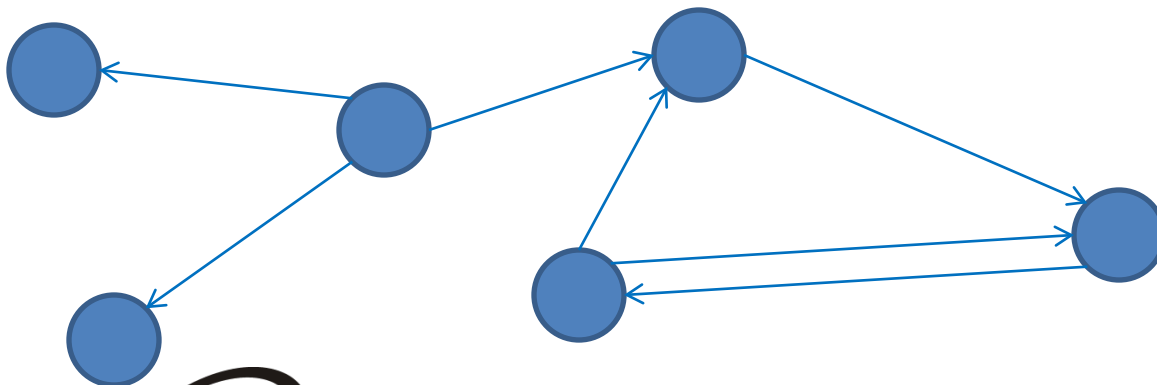
## Model for **topology** in discrete micro-level model: **directed graph**

- Each node of the graph corresponds to one object of the system
- Each edge of the graph models interaction (e.g., exchange of data) between objects



## Model of **behaviour** of each node: **Stochastic automaton**

- The automaton receives **local input** from predecessor nodes.
- At each point in time, the automaton has an **internal state**.
- The automaton decides nondeterministically about the **change of state** and about the **local output** to the successor nodes.



# A Generic Method to Derive Local Interaction Strategies

- To achieve a preferred global behavior of self-organizing systems, suitable **local interaction strategies** have to be found.
- A **general method** has been developed that allows to systematically derive local interaction strategies by specifying the preferred global behavior.



# A Generic Method to Derive Local Interaction Strategies

## Generic method:

- Model the system.
- Specify the **Laplace's Demon**: One entity of the system is chosen to be the LD and is equipped with global knowledge.
  - The LDs reactions can be specified for each opponent behaviour
- System **simulation** and recording of the LD's behaviour in each time step
  - Other entities deliver input from alphabet I
  - LD reacts with output from alphabet O
  - For each time step  $t$  a state from  $(I_{t-1} \times O_t)$  is recorded





# A Generic Method to Derive Local Interaction Strategies

- The state series is converted by the **Causal-State Splitting Reconstruction** (CSSR) algorithm
  - Result: Hidden Markov Model, which approximates the Input/Output states
- The hidden Markov Model can be used as a **local interaction strategy** for the entity by considering only those transitions that match the encountered input.

The proposed method was proven to be applicable in a game-theoretic setting, where strategies in the iterated prisoner's dilemma could be obtained.



# From Micro-level to Macro-level: Quantitative Measures

To measure self-organizing properties of a system, we need to determine the quantity of information in the system.

**Entropy** of a random variable  $X$ :

$$H(X) = - \sum P(X=w) \log_2 P(X=w)$$

$H(X)$  is the average number of bits for an optimal encoding of the information of  $X$ .

With this concept we can measure for each point of time

- the information in the whole **system**
- the information on the **internal edges**
- the information on the **input edges**
- the information on the **control edges**
- the information on the **output edges**

**Quantitative measures** can be obtained (partly based on information entropy):

Levels of

- **emergence**
  - How many globally coherent patterns are induced by local interactions?
- **autonomy**
  - How much control data from external entities are needed to keep the system running?
- **target orientation**
  - Is the high level goal, that the system designer had in his mind, reached by the system?
- **adaptivity**
  - Is the high level goal still reached after changes in the environment?
- **resilience**
  - Is the high level goal still reached after unexpected impacts on the system (e.g. break down of nodes, attacks by an intruder, ...)?
- **homogeneity**
  - Do all nodes have the same behavior?
- **global state awareness**
  - How much information does a single node have about the global state?



To measure the **level of emergence**

$$\varepsilon \in [0, 1]$$

of a system, we compute the dependencies between the values on the edges  $e \in E$ .

At time  $t$  we compare the information contained in all edges with the sum of the information contained in each single edge:

$$\varepsilon_t = 1 - (H(\text{values on all edges}) / \sum_{e \in E} H(\text{value on edge } e))$$

Level of **emergence** of the whole system  $S$ :

$$\varepsilon(S) = \text{Avg}(t \mapsto \varepsilon_t), \quad \text{where Avg is the average value of the map}$$

$\varepsilon \approx 1$     high level of emergence    (many dependencies)

$\varepsilon \approx 0$     low level of emergence    (few dependencies)

To measure the **level of autonomy**

$$\alpha \in [0, 1]$$

at time  $t$ , we compare the information contained in the **control edges**  $c \in C$  with the information contained in **all edges**  $e \in E$  during the whole run of the system:

$$\alpha_t = 1 - (H(\text{values on } C) / H(\text{values on } E))$$

Level of **autonomy** of the whole system  $S$ :

$$\alpha(S) = \text{Avg}(t \mapsto \alpha_t)$$

$\alpha \approx 1$	high level of autonomy	few control data
$\alpha \approx 0$	low level of autonomy	much control data

# Target orientation

Before a new system is designed, we have a **goal** of the system in our mind:  
The system should fulfill a given purpose.

In the model, the goal can be described by a **valuation** of configurations:

$$b : \text{Conf} \rightarrow [0, 1] \quad (\text{Conf is the set of all global states})$$

Level of **target orientation** at time  $t$ :

$$TO_t = \mathbf{E}(b(\text{Conf}_t)), \quad \text{where } \mathbf{E} \text{ is the mean value of the random variable}$$

Level of **target orientation** of the whole system  $S$ :

$$TO(S) = \text{Avg}(t \mapsto TO_t)$$

$TO(S) \approx 1$  means that the system runs through many good configurations  
) high level of target orientation

$TO(S) \approx 0$  means that the system runs through many bad configurations  
) low level of target orientation

There are different forms of **resilience** for computer networks:

- Resilience with respect to **malfunctioned nodes**
- Resilience with respect to **attacks** by an intruder, who is **inside** the network
- Resilience with respect to **attacks** by an intruder, who is **outside** the network
- Resilience with respect to **natural disasters** or other external influence, which might cause a breakdown of some nodes

How can we define Resilience in the model?



**Idea:** Replace the automata of the malfunctioned nodes by **new automata**.

If the behavior of a malfunctioned node  $v$  is not known in advance, use a **parameter**  $\theta$  to define different possible behaviors for the new automaton.

The **new goal** can be described by a valuation of configurations in the modified system  $S_\theta$ :

$$b_\theta : \text{Conf} \rightarrow [0, 1]$$

Level of **resilience** at time  $t$ :

$$\text{Res}_t = E(b_\theta(\text{Conf}_t))$$

Level of **resilience** of the whole system  $S$ :

$$\text{Res}(S) = \text{Avg}(t \mapsto \text{Res}_t)$$

$\text{Res}(S) \approx 1$  means, that the new system runs through many good configurations ) high level of resilience

$\text{Res}(S) \approx 0$  means, that the new system runs through many bad configurations ) low level of resilience



A system is **adaptive**, if it can fulfill its task despite of changes in the environment.

**Control edges:**  $C$

**Controlling nodes** of the environment:  $C^- = \{ v \in V \mid (v, w) \in C \text{ for some } w \in V \}$

**Change in the environment:** Replace the automata of  $C^-$  by new automata

If the new behavior of a node  $v \in C^-$  is not known in advance, use a **parameter**  $\theta$  to define different possible behaviors for the new automaton.

The **new goal** can be described by a valuation of configurations in the modified system  $S_\theta$ :

$$b_\theta : \text{Conf} \rightarrow [0, 1]$$

Level of **adaptivity** at time  $t$ :

$$Ad_t = E(b_\theta(\text{Conf}_t))$$

Level of **adaptivity** of the whole system  $S$ :

$$Ad(S) = \text{Avg}(t \mapsto Ad_t)$$

$Ad(S) \approx 1$  means, that the new goal is reached despite the changes in the environment (the new system runs through many good configurations)

$Ad(S) \approx 0$  means, that the goal is not reached after the changes (the new system runs through many bad configurations)



# Homogeneity

How can we measure **homogeneity**?

Consider the **local view** of a node  $v$ :

At the current point of time  $t$ , the node  $v$  sees not the whole configuration  $Conf_t$  but only the local configuration  $Conf_{t,v}$ , that is visible at  $v$ : The internal state of  $v$  and the values of the edges of  $v$ .

Level of **homogeneity** at time  $t$ :

$$Ho_t = 1 - \frac{\sum_{v,w \in V, v < w} |H(Conf_{t,v}) - H(Conf_{t,w})|}{\sum_{v,w \in V, v < w} \max(H(Conf_{t,v}), H(Conf_{t,w}))}$$

Level of homogeneity of the whole system  $S$ :

$$Ho(S) = \text{Avg}(t \mapsto Ho_t)$$

$Ho(S) \approx 1$  means, that the system is homogeneous

$Ho(S) \approx 0$  means, that the system not homogeneous at all

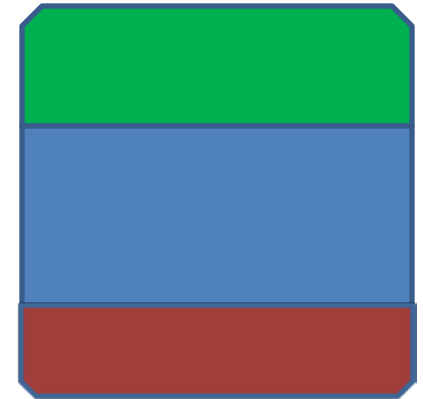
To measure the **level of global state awareness**

$$\omega \in [0, 1]$$

the initial states are partitioned according to the equivalence relation induced by a property of interest.

- Measurement of the information about the initial equivalence class inside of each node:

$$\omega_t = 1 - \frac{H(\text{equivalence class} \mid \text{local history})}{H(\text{equivalence class})}$$



Level of **global state awareness** of the whole system  $S$ :

$$\omega(S) = \text{Avg}(t \mapsto \omega_t)$$

$\omega \approx 1$  means high level of global state awareness

(each node knows much about initial equivalence class)

$\omega \approx 0$  means low level of global state awareness

(each node knows few about initial equivalence class)

## ■ Problem: **Scalability**

- In large systems, it might be difficult to compute the quantitative measures.
- **Simulation** results may be used to get approximations.
- For entropy based measures, the probabilities can be approximated by the **relative frequencies** of the events in the simulation runs.
- Mean values of random variables can be approximated by the **arithmetic mean** values of the outcome of the variables in the simulation runs.

- How can the quantitative measures be used to provide a framework to study how global phenomena emerge in complex self-organizing systems from local interactions?
- How can the quantitative measures help to improve the design of new complex systems?



# Example: Slot synchronizing in wireless networks

## Wireless network:



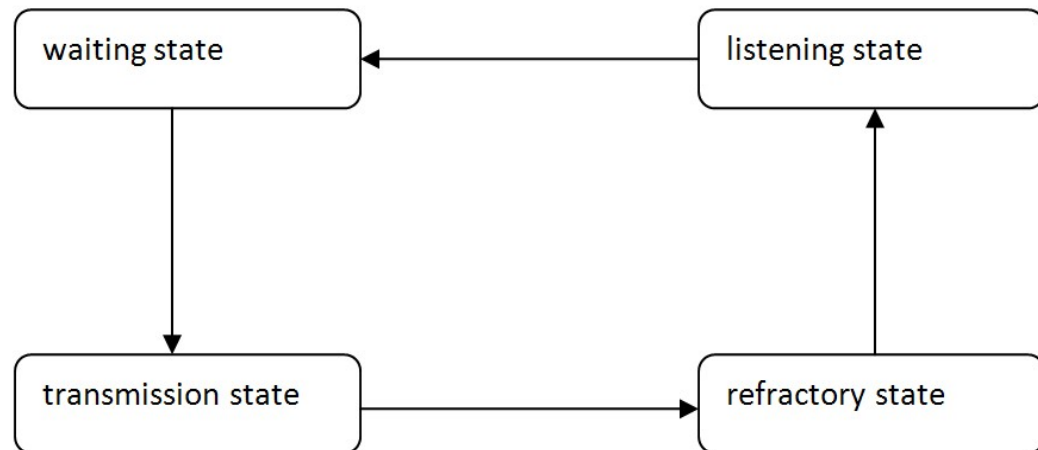
- For communication, time is divided into slots.
- There is no central clock, which defines when a slot begins.
- The nodes try to synchronize the slots.

# Example:

## Slot synchronizing in wireless networks

### Slot synchronization algorithm of Tyrrell, Auer and Bettstetter:

- At each point of time, each node is in one of four different states:
- In the **transmission state**, the node transmits a pulse to it's neighbors to indicate the beginning of a slot.
- In the **listening state**, the node can receive and decode pulses from it's neighbors and it adjusts its phase function  $\phi$  according to these pulses. The listening state ends, when the threshold  $\phi_{\max} = 1$  is reached.
- In the **waiting state** and in the **refractory state**, the node does nothing.
- The length of an uncoupled cycle is  $2T$  with  $T > 0$ .



# Example:

# Slot synchronizing in wireless networks

## Simulation results show:

- **Two groups of synchronizations** are built.
- Inside each group we have good synchronization: Each object of the group fires a pulse at nearly the same time like the other objects of the group.
- The second group fires  $T$  time units after the first group.
- By using slots of length  $T$  we get a good slot synchronization

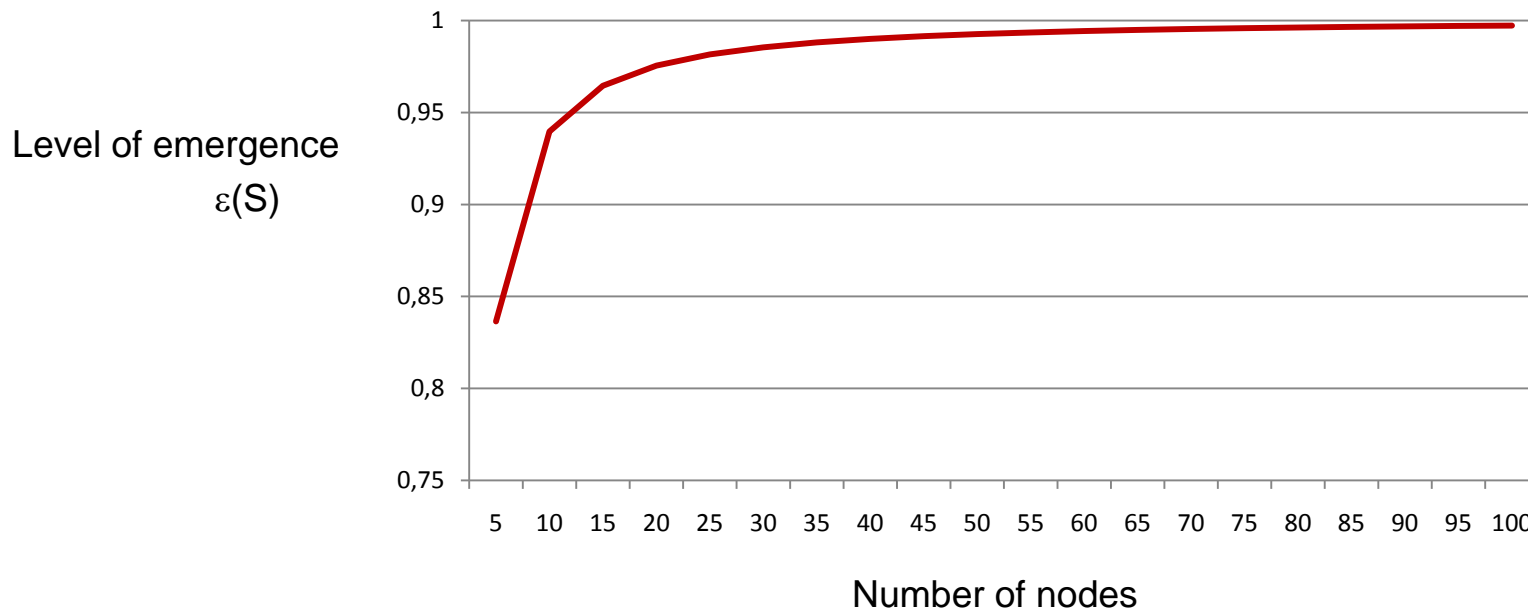




# Example: Slot synchronizing in wireless networks

The system is **autonomous** since it does not contain external node:  $\alpha = 1$

The synchronization of the objects is **emergence**, since this is a global property of the system, which is induced by the local interactions.



# Target orientation

Calculation of the level of **target orientation**:

The **goal** is to minimize the time differences between the beginning of the slots of the nodes.

$\text{dist}_c(v,w)$ : **Slot distance** of nodes  $v, w \in V$  in configuration  $c$

$$b(c) = 1 - \frac{\sum_{v,w \in V} \text{dist}_c(v,w)}{|V^2| \cdot T/2}$$

After fixing the system parameters, we can calculate the level of target orientation.

$|V| = 30, T = 100, T_{\text{dec}} = 15, T_{\text{Tx}} = 45, T_{\text{refr}} = 35, T_{\text{wait}} = 40, \alpha = 1.2, \beta = 0.01$

**Target orientation:**  $\text{TO}(S) \approx 0.997$

For the level of **resilience**, we first have to fix the behavior of the malfunctioned nodes.

Resilience with respect to the **break down** of a node:

If the graph is still strongly connected after the break down of a node, then the remaining nodes are able to synchronize:  $\text{Res}(S) \approx \text{TO}(S) \approx 1$

Resilience with respect to an **intruder** at a node  $v_0$  periodically sending pulses to its neighbors.

$|V| = 30, T = 100, T_{\text{dec}} = 15, T_{\text{Tx}} = 45, T_{\text{refr}} = 35, T_{\text{wait}} = 40, \alpha = 1.2, \beta = 0.01$

Choose  $\theta$  as the time between two pulses of the intruder.

$\theta$	45	70	100	120	150
Res(S)	0.987	0.985	0.996	0.991	0.996

Level of **adaptivity**:

There are no external nodes in the system.

$$Ad_t = E(b_{\theta}(\text{Conf}_t)) = TO_t$$

$$Ad(S) = TO(S) \frac{1}{4} 0.997$$



Level of **homogeneity**:

For a regular graph, the system is homogeneous:

$$Ho_t = 1 - \frac{\sum_{v,w \in V, v < w} |H(Conf_{t,v}) - H(Conf_{t,w})|}{\sum_{v,w \in V, v < w} \max(H(Conf_{t,v}), H(Conf_{t,w}))} = 1$$

$$Ho(S) = 1$$

If the degree of the nodes differ, the system is not fully homogeneous, but it reaches a high level of homogeneity after building the synchronization groups:  $Ho(S) \approx 1$



# Example: Evacuation scenario

- Scenario: Evacuation in a building
  - Each person wears a life belt:
    - Ambient Intelligence (Aml) device, which is able to communicate with other life belts to improve the evacuation.



# Example: Evacuation scenario

- Topology
  - Each node in the graph represents one person wearing a life belt.
  - Each edge in the graph represents a communication channel
  - Since the persons move around, the topology changes during time
    - ) Graph  $G_t = (V, E_t)$  depends on time  $t$
  - In a simple scenario, external nodes are not needed. The system will self-organize.
    - In a more complex scenario, external nodes can be introduced to model changes in the environment (e.g. break down of a part of the building, etc.)



# Example: Evacuation scenario

- Behaviour
  - The behaviour of each node is represented by an automaton.
  - Internal state contains
    - Current position
    - Some relevant information about exits received by other nodes in the past.
  - Local rules for state change and communication with neighbours:
    - **Problem:** How can we find **good** rules to maximize the evacuation?
    - **Idea:** Define different rule sets (or a fixed rule set containing some global rule parameters) and use quantitative measures for comparison.





# Example: Evacuation scenario

For the target orientation we need a **valuation** of configurations:

$b : \text{Conf} \rightarrow [0, 1]$

- In the evacuation scenario the good configurations are those where many people have already escaped:
  - $b(c) = \text{\#escaped}/N$   $N = \text{number of persons}$
- Consider a run of the system starting at time  $t = 0$  ending at  $t = T$ .
- $TO_t = E(b(\text{Conf}_t))$  is a nondecreasing function
- Goal: Try to maximize  $TO_T$
- Different rule sets can be compared.
- Rules with different parameters can be compared.



# Example: Evacuation scenario

- Next to the target goal orientation also other measures might be useful to **compare different strategies** and/or **different rule parameters**.
- Example: **Level of global state awareness**



# Example: Evacuation scenario

To measure the **level of global state awareness**

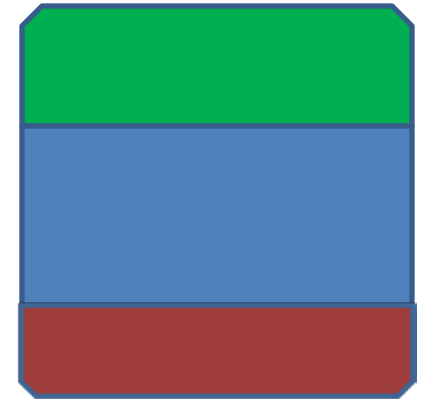
$$\omega \in [0, 1]$$

The initial states are partitioned according to the equivalence relation induced by a property of interest.

- Measurement of the information about the initial equivalence class inside of each node:

$$\omega_t = 1 - \frac{H(\text{equivalence class} \mid \text{local history})}{H(\text{equivalence class})}$$

- Equivalence class 1:
  - All configurations where exit 1 has highest throughput
    - high capacity, only small crowd near the exit
- Equivalence class 2:
  - All configurations where exit 2 has highest throughput
- Equivalence class 3:
  - All configurations where exit 3 has highest throughput



# Example: Evacuation scenario

- **Idea:** Each person tries to learn in which equivalence class the system is and uses the information about his own position to decide which exit fits best for him.
- The more information the people get about the equivalence class the better they can decide about the best exit.
- Communication between the nodes is used for **learning**:
  - “I found an exit at position  $(x, y)$ ”
  - “At my current position  $(x, y)$  are  $k$  other persons in the neighbourhood.”
  - ...



# Example: Evacuation scenario

- Calculate the measure of **global state awareness**  $\omega(S)$  for different strategies.
- The strategy with highest global state awareness yields the **maximal information** such that each person has good chances that his decision is really best.
- **Problem:**
  - Before the level of global state awareness can be applied, a point of time T for the **initialization** must be fixed.
  - The equivalence classes always belong to this point of time T.
  - Starting the system at time T, the people learn in each step something about the equivalence class at time T.
  - The best exit at time  $t > T$  might differ from the best exit at time T.
  - But this method is still useful since the information spread is much faster than the movement of the persons.



# Conclusion

The mathematical modeling can be used for **a wide variety of systems**:

- **Technical systems**
- **Biological systems**
- **Physical systems**

and many more.

The models can help to analyse the behavior of complex systems.

**Quantitative measures** provide a link from the micro level to the macro level:

- They describe **global properties of the system**
- They can be used for the **analysis** of real world systems.
- They can be used for **design, engineering** and **optimization** of new systems.



**Thank you for your attention**

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