

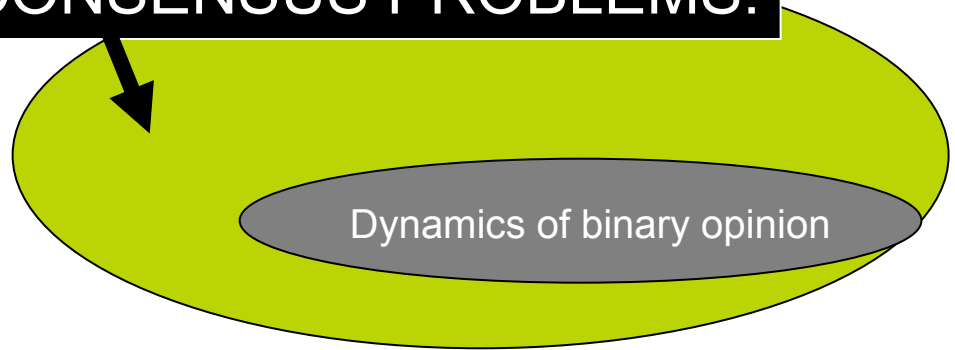
SOCIAL CONSENSUS AS A SELFORGANIZATION PHENOMENON

MAXI SAN MIGUEL



Determine when and how the dynamics of a set of interacting units (agents) that can choose among several **options** (*political vote, opinion, cultural features,...*) leads to a **consensus** in one of these opinions, or when a state with several **coexisting** options prevails.

CONSENSUS PROBLEMS:



INTERACTIONS: Mechanisms (“rule”) and Network (with whom)

-VOTER MODEL

-SPIN FLIP KINETIC ISING MODEL (T=0)

-AXELROD MODEL

-GRANOVETTER’S MODEL

MODELS

-Imitation

-Following majority. Social pressure

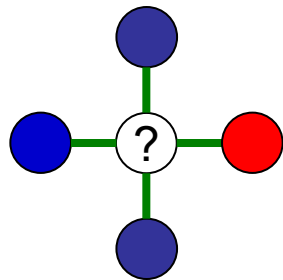
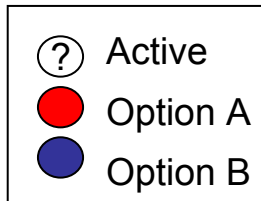
-Homophily

-Threshold for social pressure

MECHANISMS

MODELS of CONSENSUS with **TWO OPTIONS** :

- Prototype models *with excluding options*: - VOTER MODEL
- SPIN FLIP KINETIC ISING MODEL T=0



$$p_{? \rightarrow B} = 3/4$$

$$p_{? \rightarrow A} = 1/4$$

$$p_{? \rightarrow B} = 1$$

$$p_{? \rightarrow A} = 0$$

Voter Model

RANDOM IMITATION

Spin Flip
Kinetic Ising T=0

SOCIAL PRESSURE

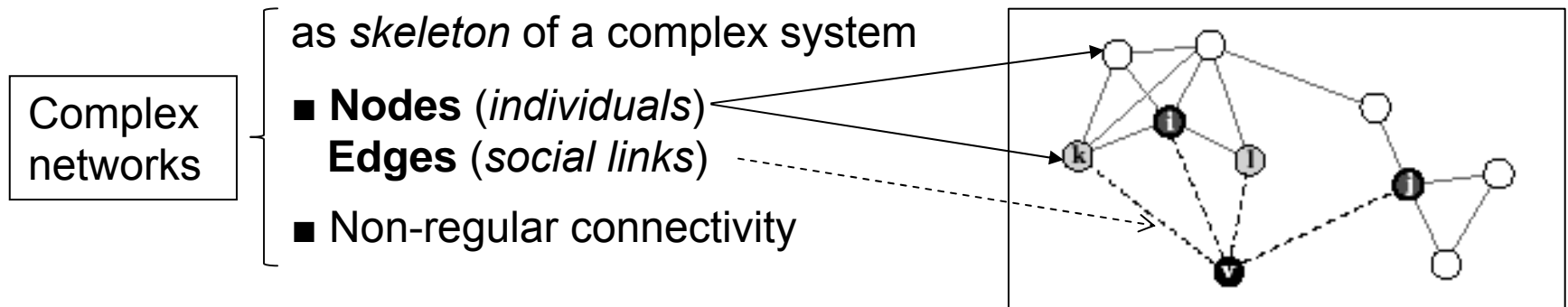
- New issue/class of models: **AB agents with coexisting options**

Example: Bilingual agents in the dynamics of two competing languages

General: Coexistence of social norms at the individual level (linux or windows)

Complex Networks

Albert, Barabási *Rev. Mod. Phys.* **74**, 47 (2002)

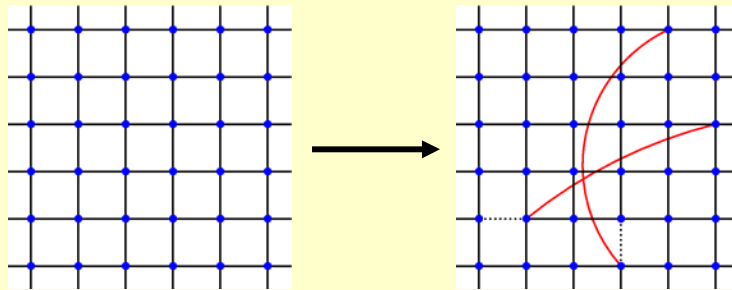


Modelling of: biological, technological, social systems

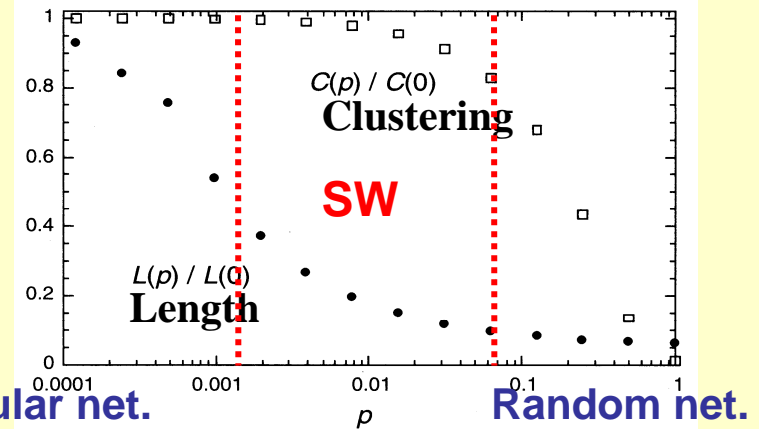
- ▶ Main contributions concern
 - **Small World Phenomenon**
 - **Scale-Free Networks**
 - **Mesoscale Structure: communities**

Small World Networks

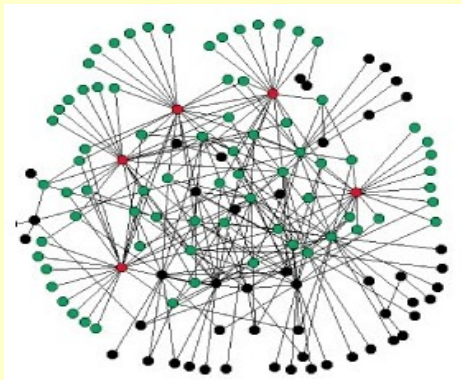
Watts-Strogatz, Nature **393**, 440 (1998)



Rewire with prob. p



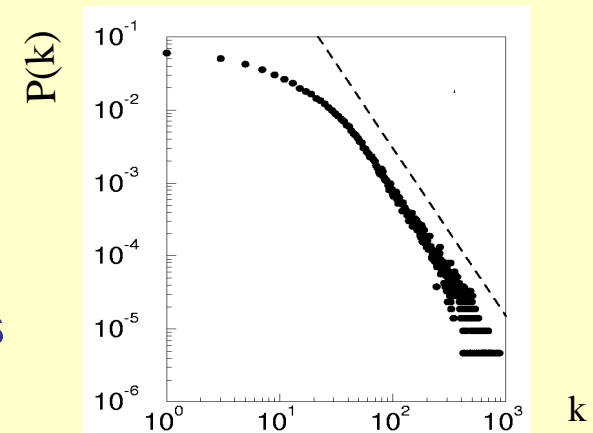
Barabasi-Albert Scale Free Networks



Power law for the degree distribution

$$P(k) \sim k^{-\gamma}, \gamma=3$$

Importance of hubs



Albert & Barabasi, Rev. Mod. Phys. **74**, 47 (2002)

Ingredients of interacting agents models:

a) Mechanism: Voter model (imitation)

b) Who interacts with whom? Tie heterogeneity

Complex networks

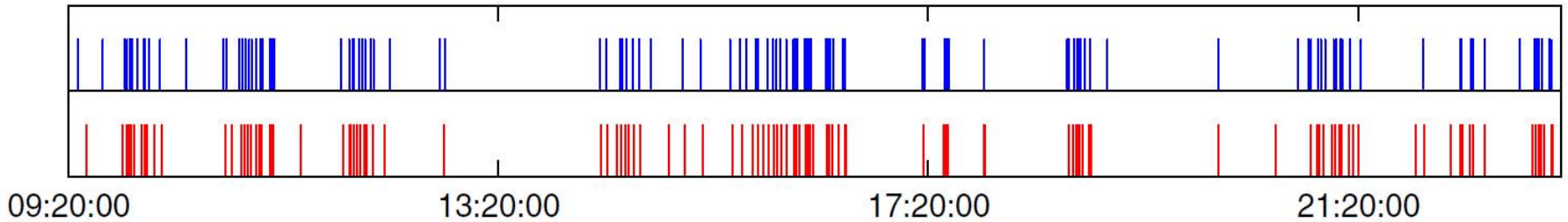
Co-evolution : Ties are not persistent

 c) When do elements interact? Interaction activity

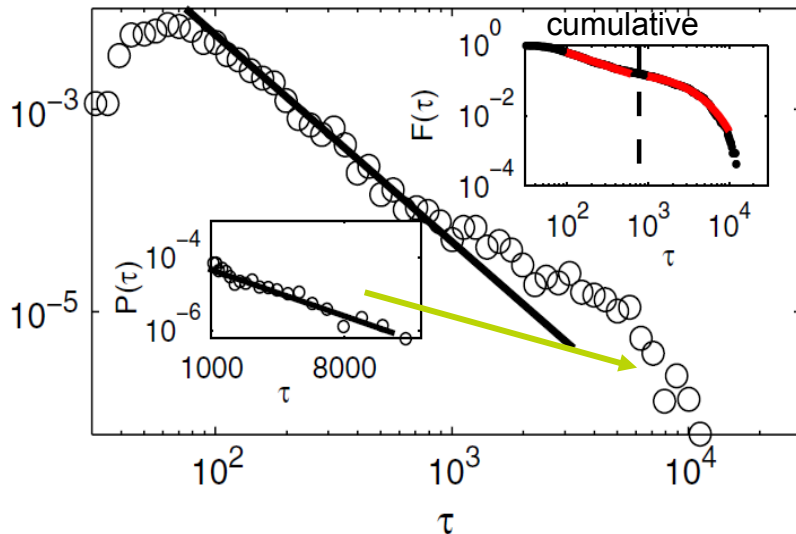
Constant rate

or

Temporal Heterogeneity



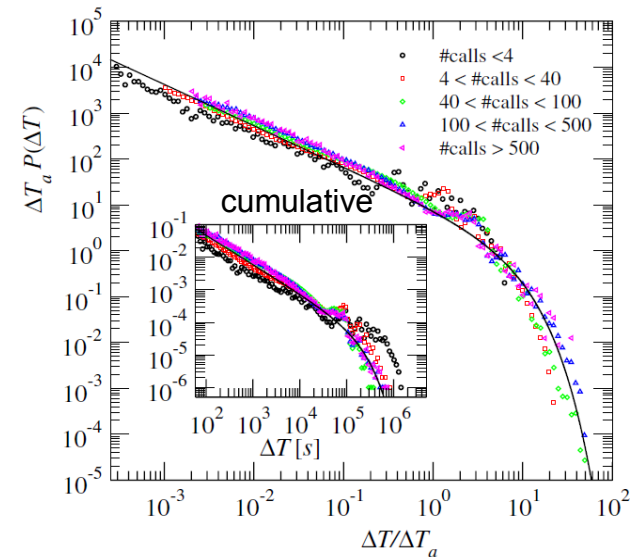
Bursty pattern in SMS sending. (Ye Wu et al. PNAS 107, 18803, 20010)



Interevent distribution for SMS
(Ye Wu et al. PNAS 107, 18803 (2010))



Heavy tailed distributions



Interevent time distribution for calling activity (J.Candia et al. J. Phys. A: Math. Theor. 41, 224015 (2008))

Origin vs consequences

* **ROLE OF COMPLEX STATIC NETWORKS (VOTER MODEL)**

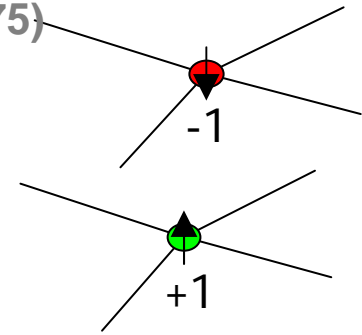
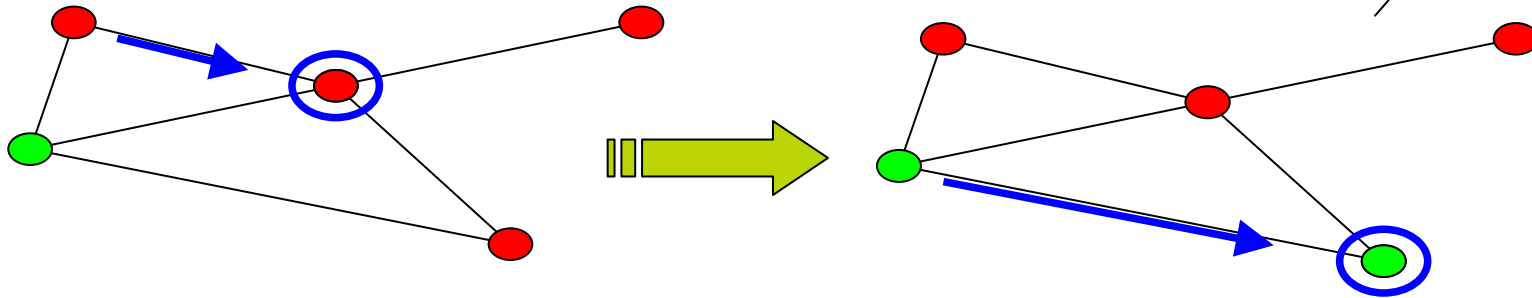
AXELROD MODEL OF CULTURAL DYNAMICS:

* **LINK DYNAMICS: COEVOLUTION**

* **SELFORGANIZATION VS IMPOSED ORGANIZATION**

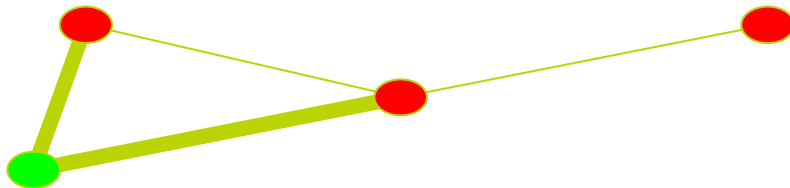
Ann. Probability (1975)

- “Voters” located in the nodes of a network have “opinions” $\sigma_i=1$ or $\sigma_i=-1$.
- **A randomly chosen voter takes the opinion of one of its neighbors (node update).**



*Qs?: When and how one of the two absorbing states (**consensus**) is reached? Effect of network of interactions?*

Order Parameter: Average interface density



$$\rho = \frac{1}{2N\langle k \rangle} \left(\sum_{i=1}^N \sum_{j \in \nu(i)} (1 - \sigma_i \sigma_j) \right)$$

$\rho=0$ in absorbing state

Interface: a link connecting nodes with different states.



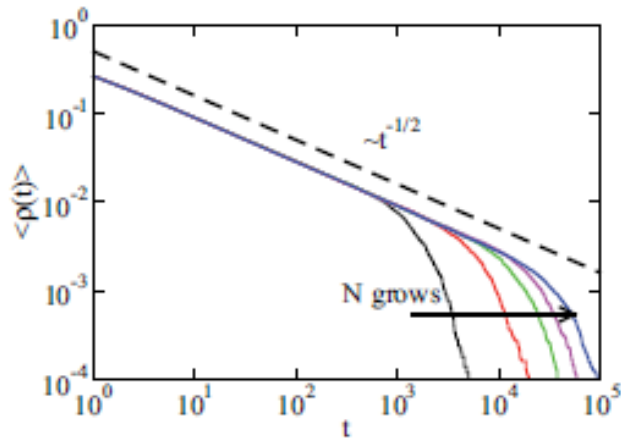
Voter Model in regular networks

$$\langle \rho \rangle \sim \begin{cases} t^{-1/2}, & d = 1 \\ (\ln t)^{-1}, & d = 2 \\ \xi - bt^{-d/2}, & d > 2 \end{cases} \quad \tau \sim \begin{cases} N^2, & d = 1, \text{ time to reach absorbing state} \\ N \ln N, & d = 2, \text{ time to reach absorbing state} \\ N, & d > 2, \text{ survival time of metastable state} \end{cases}$$

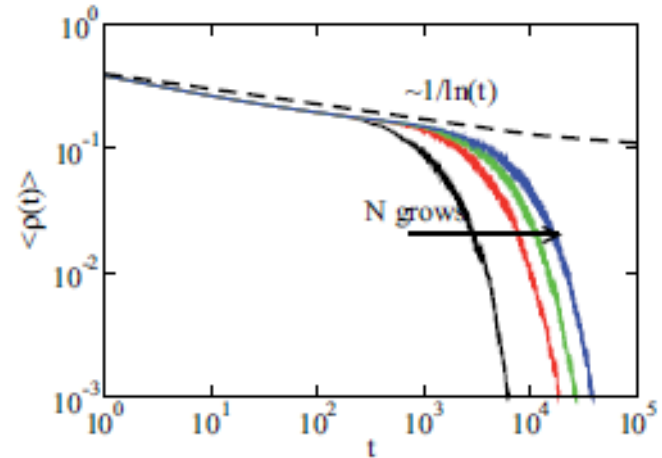
d=1,2: Ordering

Unbounded growth of domains of absorbing states

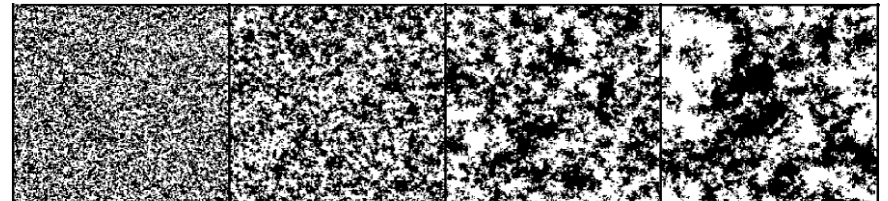
d=1



d=2



*Coarsening without surface tension:
Driven by interfacial noise*



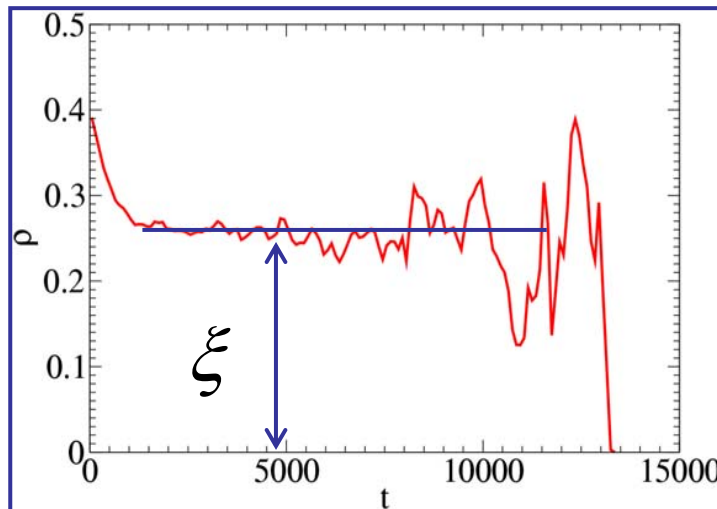
d>2 regular and complex networks

$$\langle \rho \rangle \sim \xi$$

$\tau(N) \approx N$, survival time of metastable state

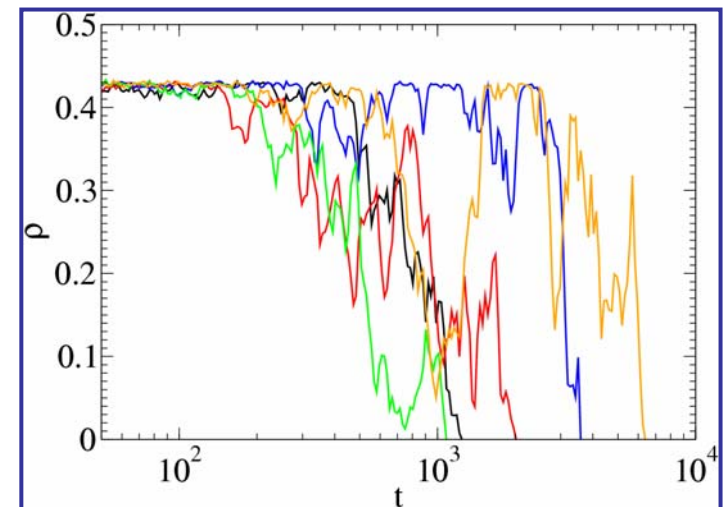
d>2: No Ordering: Dynamical Metastability

Disordered states.



$l = \xi^{-1}$ Characteristic size of ordered domain

Finite size fluctuations take the system to an absorbing state



$\langle \rho \rangle \sim e^{-t/\tau}$ τ survival time

N-state voter model in 2d lattice



Voter Model in Complex Networks

http://ifisc.uib-csic.es/research/applet_complex/Voteraplet/applet.html



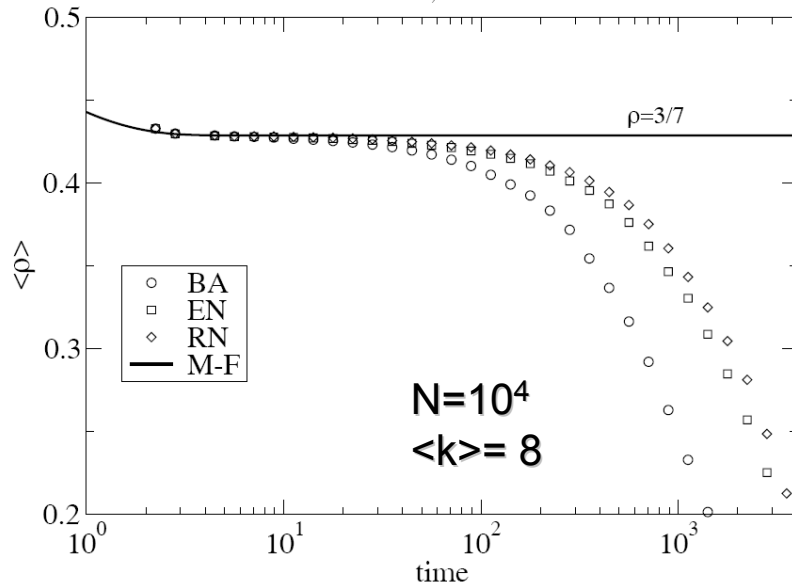
Mean Field Link Dynamics:

$$\rho^s = \xi = \frac{\langle k \rangle - 2}{2(\langle k \rangle - 1)}$$

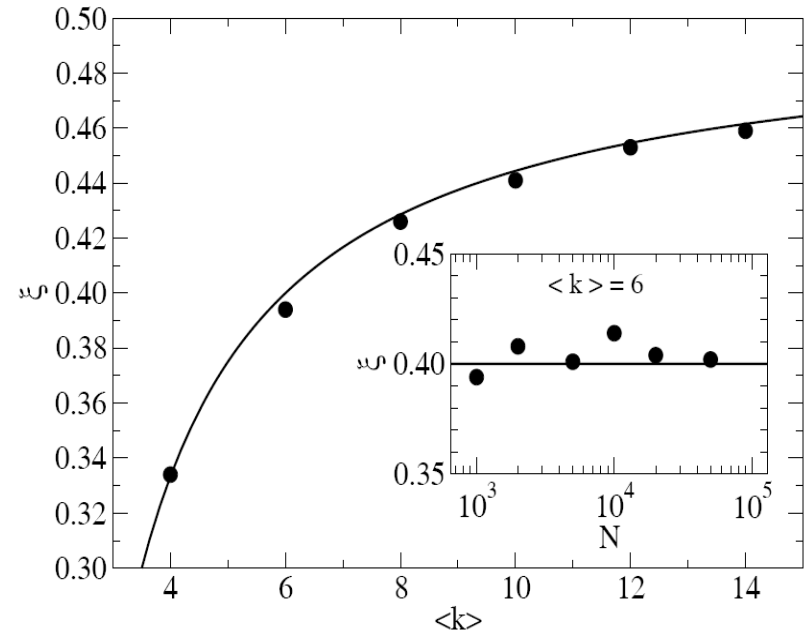
Single parameter theory

$$\tau = \frac{(\langle k \rangle - 1) \langle k \rangle^2}{2(\langle k \rangle - 2) \langle k^2 \rangle} N$$

Network topology independence



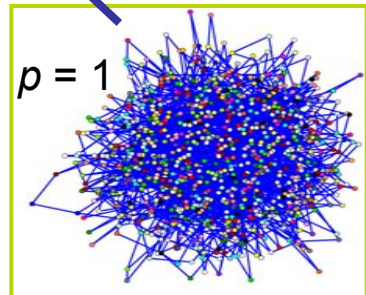
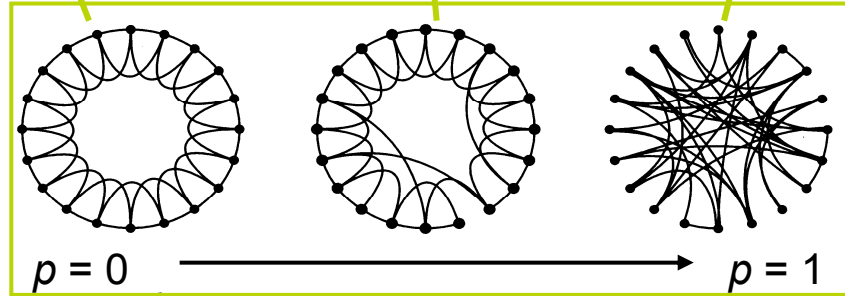
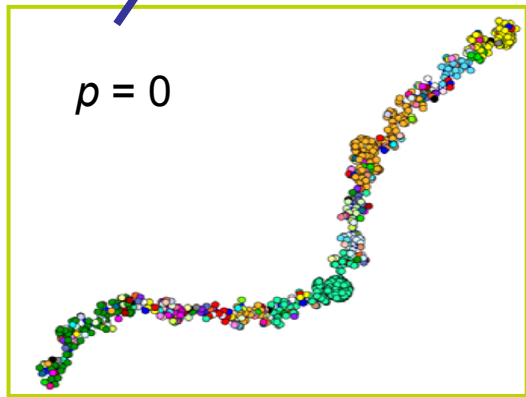
Barabasi-Albert Scale Free Networks



Network Disorder and Link Heterogeneity

Disorder: Rewiring parameter $0 < p < 1$. $d=1 \longrightarrow$ random networks

$p = 0$	$0 < p < 1$	$p = 1$	
1D regular	Small-World: SW	Random: RN / EN	<u>Single SCALE</u>
Structured SF: SSF	Small-World SF: SWSF	Random SF: RSF / BA	<u>HUBS</u> $P(k) \sim k^{-3}$
$d = 1$	$d = \infty$		



$P(k) \sim k^{-3} ; L \sim N$

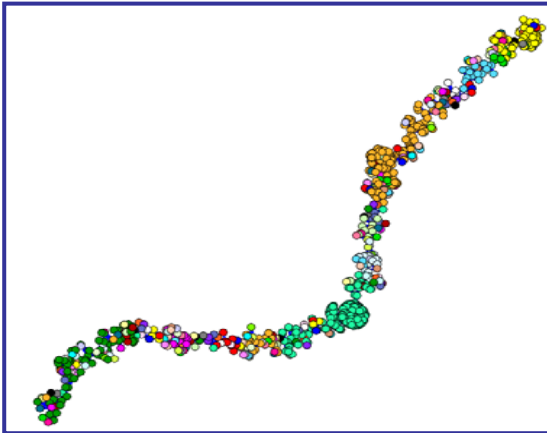
$P(k) \sim k^{-3}$

Role of dimensionality

1D Scale free net?

Structured SF: **SSF**

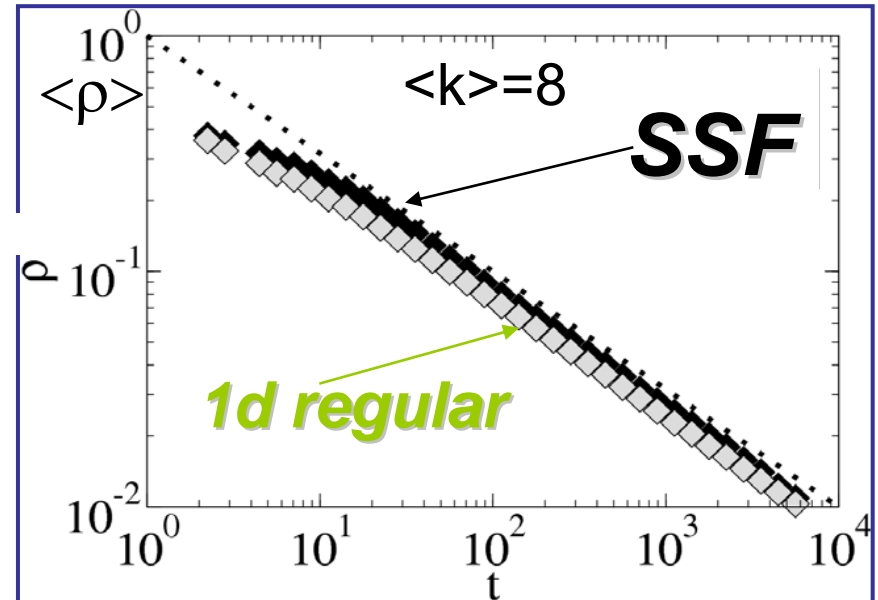
Klemm and Eguíluz,
Phys. Rev. E **65**,036123 (2002)



Scale free but
high clustering and 1d

$$P(k) \sim k^{-3}$$

$$L \sim N \quad C \sim N^0$$



$$\text{SSF} \quad \langle \rho \rangle \sim t^{-1/2}$$

$$\tau_1 \approx N^2$$

Dimensionality determines when voter
dynamics orders the system

Degree distribution or network **disorder**
are not relevant

Voter model dynamics in complex networks: Role of dimensionality,

Suchecki, Krzysztof; Eguíluz Víctor M.; San Miguel, Maxi
Physical Review E **72**, 036132(1-8) (2005)

Conservation laws for the voter model in complex networks

Suchecki, Krzysztof; Eguiluz, Victor M.; San Miguel, Maxi
Europhysics Letters **69**, 228-234 (2005)

Generic absorbing transition in coevolution dynamics

Vazquez, F.; Eguiluz, V. M.; San Miguel, M.
Physical Review Letters **100**, 108702 (1-4) (2008)

Analytical Solution of the Voter Model on Uncorrelated Networks

Vazquez, F.; Eguiluz, V. M.
New Journal of Physics **10 No.6**, 063011 (1-19) (2008)

Conservation laws for voter-like models on random directed networks

Serrano, M. Ángeles; Klemm, Konstantin; Vazquez, Federico; Eguíluz, Víctor M.; San Miguel, Maxi
Journal of Statistical Mechanics: Theory and Experiment , P10024 (2009)

Agent Based Models of Language Competition: Macroscopic descriptions and Order-Disorder transitions

Vazquez, Federico; Castello, Xavi; San Miguel, Maxi
Journal of Statistical Mechanics: Theory and Experiment **2010**, P04007 (2010)

Update rules and interevent time distributions: Slow ordering vs. no ordering in the Voter Model

Fernández-Gracia, Juan; M. Eguíluz, Víctor; San Miguel, Maxi
arXiv: [1102.3118](https://arxiv.org/abs/1102.3118), Phys. Rev E (2011)

A measure of individual role in collective dynamics: spreading at criticality

Klemm, Konstantin ; Serrano, M. Angeles; Eguiluz, Victor M. ; San Miguel, Maxi
arXiv [1002.4042](https://arxiv.org/abs/1002.4042) (2011)

Dynamics of Networks:

1. Dynamics **OF** network formation: Structure created by individual choices/actions
2. Dynamics **ON** the network: Actions of individuals constrained by the social network

Rightwing view



Leftwing view



3. Co-evolution of agents and network :

Circumstances make men as much as men make circumstances

..new research agenda in which the structure of the network is no longer a given but a variable.....explore how a social structure might evolve in tandem with the collective action it makes possible (Macy, Am. J. Soc. 97, 808 (1991))

Final Goal: Understanding dynamical processes of group formation and social differentiation: Emergence of social dynamical networks with

- Social structure
- Weak links (Granovetter)
- Community structure

Emergence: (P.W. Anderson, Science **177**, 393 (1972))

“The reductionist hypothesis does not by any means imply a constructionist one”

Sociology can not be reduced to psychology as molecular biology is not applied chemistry: “At each level of complexity entirely new properties appear”

Examples of emergence: Traffic from cars, clustering in residential segregation,
V shape of bird flocks, psychohistory.....

What is distinctive of emergence in human social systems?

-Downward causation goes further in human societies

-Second-order emergence:

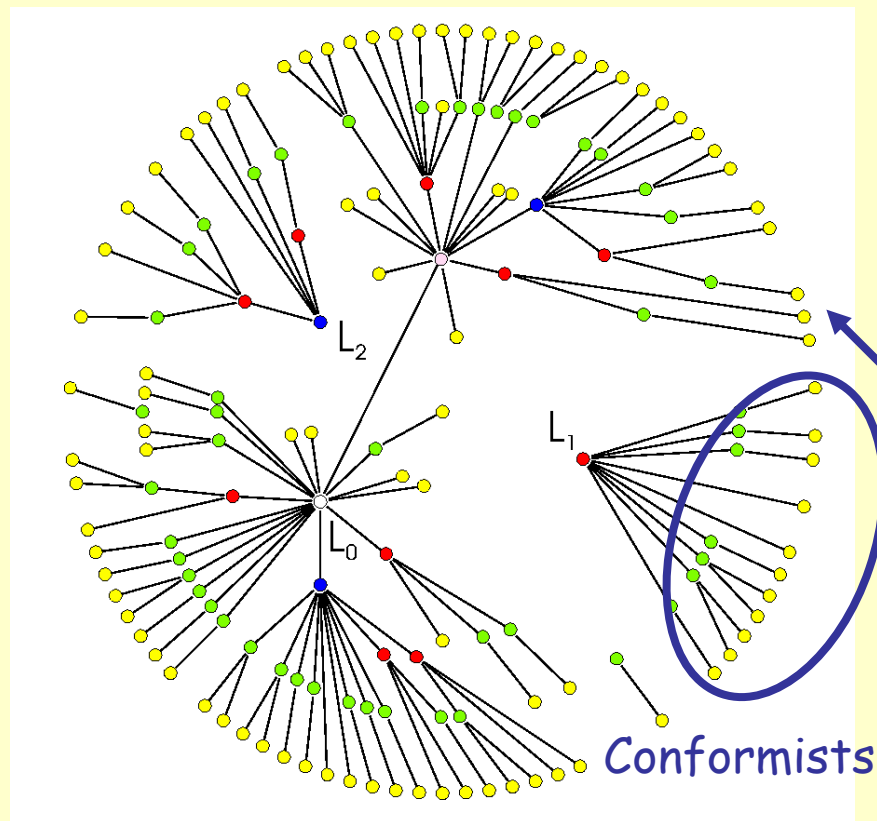
Humans can recognise and react to the emergent global structure

- Individual action leads to emergent social structures
- These structures are the matrix in which action takes place
- This action maintains and changes the structures

Spatial Prisoner's Dilemma Game: Cooperation maintained by local interactions

(M. A. Nowak and R. M. May, *Nature* 359, 826 (1992); B. Huberman and S. Glance, *PNAS* 90, 7716 (1993))

Network Dynamics (Choosing partners): Unsatisfied Defectors break (probability p) any link with neighbouring Defector and establishes a new link in the network



Social differentiation: Emergence of

Leaders

Conformists

Exploiters

Imitation network of Cooperators

Absolute leader L_0 :

Largest pay-off in the network
and

largest number of links

Review paper: *T. Gross and B. Blasius, J. R. Soc. Interface 5, 259 (2008)*

Key ingredients.

a) Going beyond dynamical models in which:

- Network evolution is decoupled from the evolution of agents actions
- Complete network redefined at each time step

b) Social plasticity as ratio of time scales of evolution of network and action



Generic result: Network fragmentation transition

(Independent of link conservation, rewiring rule, interaction....)

Zachary's karate club

Two examples in model of consensus dynamics:

* **Voter model:** Minimal model

F. Vázquez, V. M. Eguíluz and M. San Miguel, Phys. Rev. Lett. 100, 108702 (2008)

* **Axelrod's cultural model:** Robustness of globalization-polarization transition

F. Vazquez et al. Physical Review E, 76, 046120 (1-5) (2007)
D. Centola et al. Journal of Conflict Resolution, 51, 905-929 (2007)

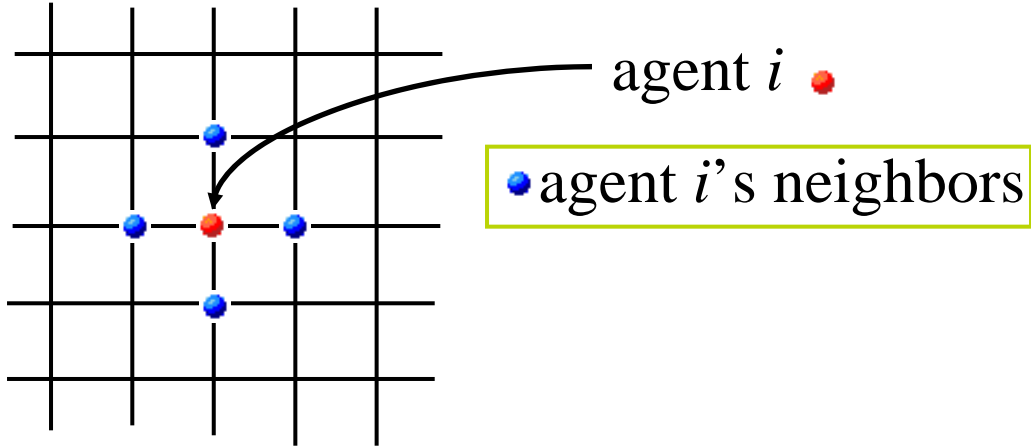
Proposal: Model to explore mechanisms of competition between ***globalization*** and persistence of ***cultural diversity ("polarization")***

Definition of culture: Set of individual attributes subject to social influence

Principle of Homophily: Promotes interaction between similar.
"like attracts like"

Principle of Social Influence: Promotes cultural similarity. *The more two interact the more similar they become.*

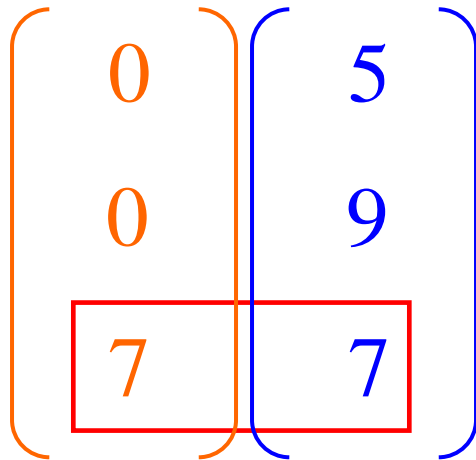
Axelrod's conclusion: Combination of homophily and social influence produces and sustains polarization (cultural diversity)



$$\begin{pmatrix} \sigma_{i1} \\ \sigma_{i2} \\ \vdots \\ \sigma_{iF} \end{pmatrix} \quad \begin{array}{l} F = \# \text{Features} \\ q = \# \text{Traits per} \\ \text{feature} \\ \sigma_{if} \in \{0, \dots, q-1\} \end{array}$$

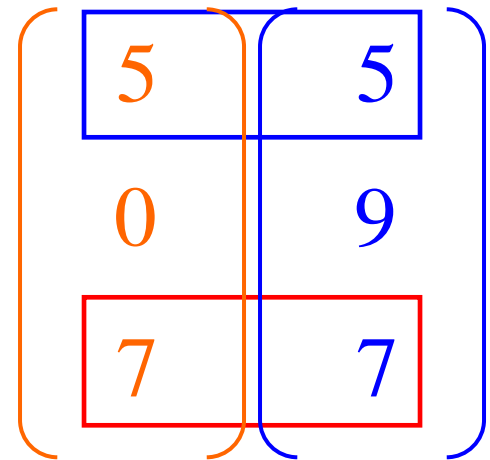
$F=3; q=10$

$q^F (10^3)$ equivalent cultural options.

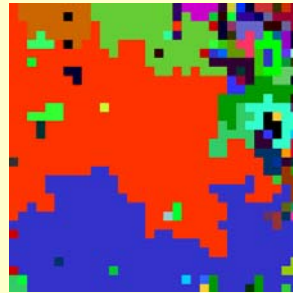
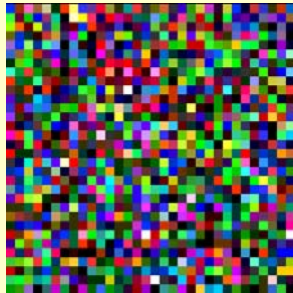


Mechanism of local convergence:

Prob to interact =

$$\frac{\text{Common features}}{F} = \frac{1}{3}$$


$$F = 3, q = 10$$



$t = 0$ →

System freezes in
an absorbing
multicultural state

[http://ifisc.uib-csic.es/
research_topics/socio/culture.html](http://ifisc.uib-csic.es/research_topics/socio/culture.html)

- The model illustrates how **local convergence** can generate **global polarization**.
- Number of domains taken as a measure of cultural diversity
- Uniform state always prevails without similarity rule (*Kennedy 1998*)

- **Order parameter:** S_{\max} size of the largest homogeneous domain
- **Control parameter:** q measures initial degree of disorder.

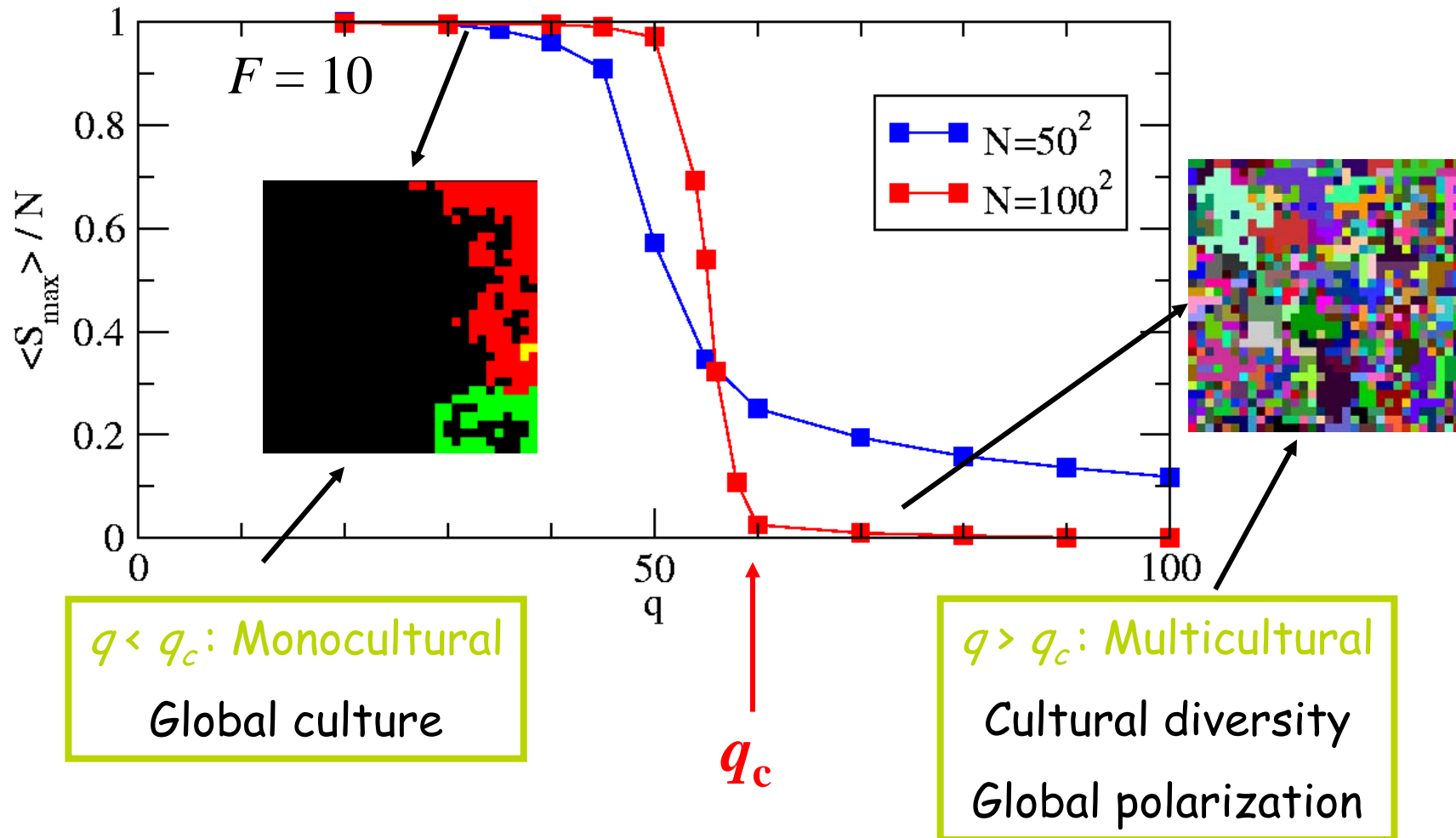
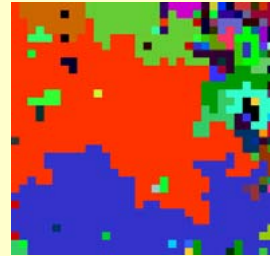
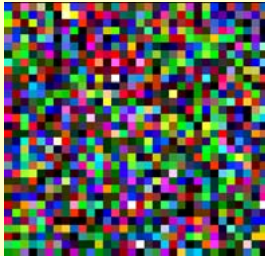
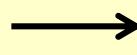


Illustration of how **local convergence** can generate **global polarization**.



$t = 0$



System freezes in an absorbing multicultural state

➔ Frozen polarized states stable?

➔ Robustness of globalization-polarization transition?

* **Cultural drift:** “Perhaps the most interesting extension and at the same time, the most difficult one to analyze is cultural drift (modeled as spontaneous change in a trait).”

R. Axelrod, J. Conflict Res. (1997)

➔ Polarized states are not stable and cultural diversity is destroyed

Klemm et al., Phys Rev. E 67, 045101R (2003); J. Economic Dynamics and Control 29, 321 (2005)

* **Coevolution:**

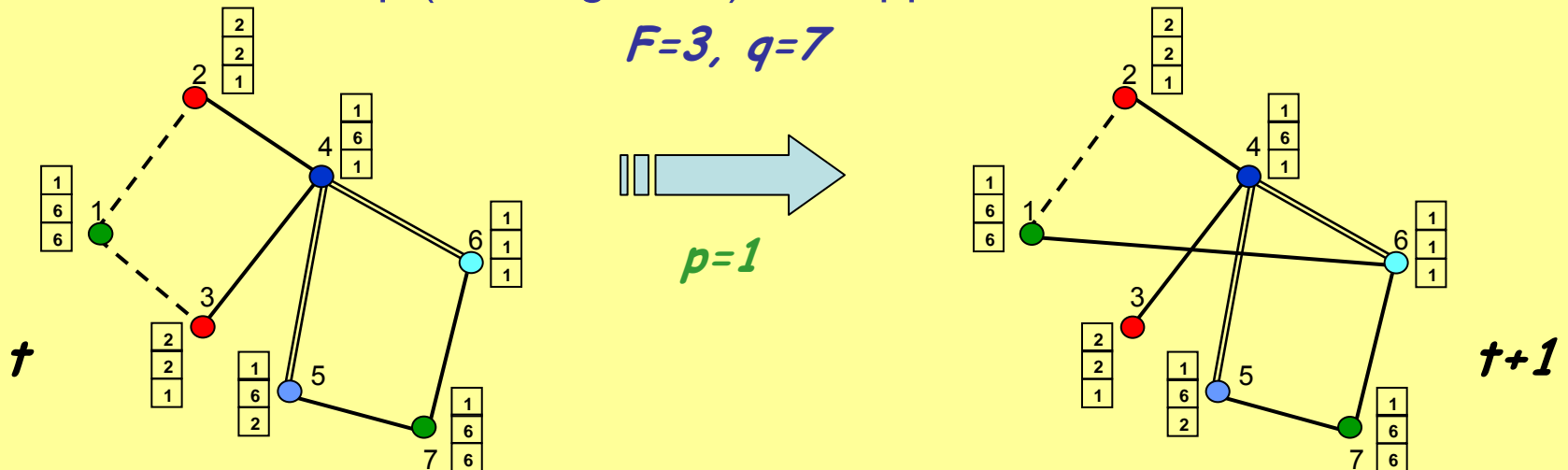
➔ New specification of homophily

➔ Transition robust. Culturally polarized states robust vs cultural drift

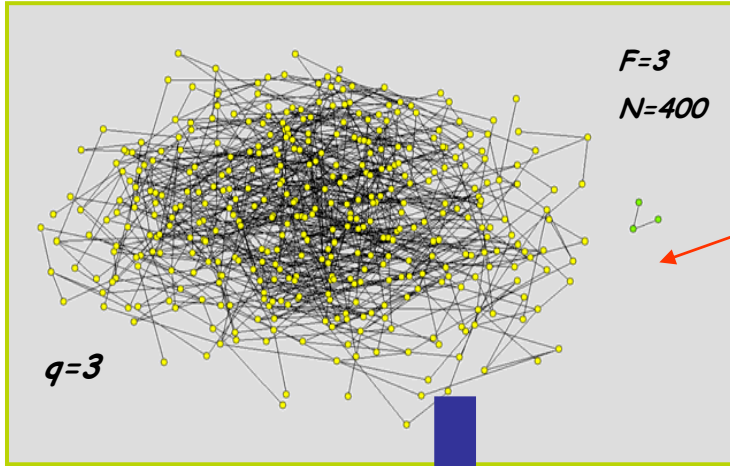
Step 1: Choose randomly a link connecting two agents and calculate the overlap (number of shared features). Probability of interaction is proportional to the overlap (if overlap is not maximum)

Step 2: Social influence dynamics: interaction results in one more common trait

Step 3: NETWORK DYNAMICS: New homophily specification
A link with zero overlap (cleavage-link) is dropped + new link established

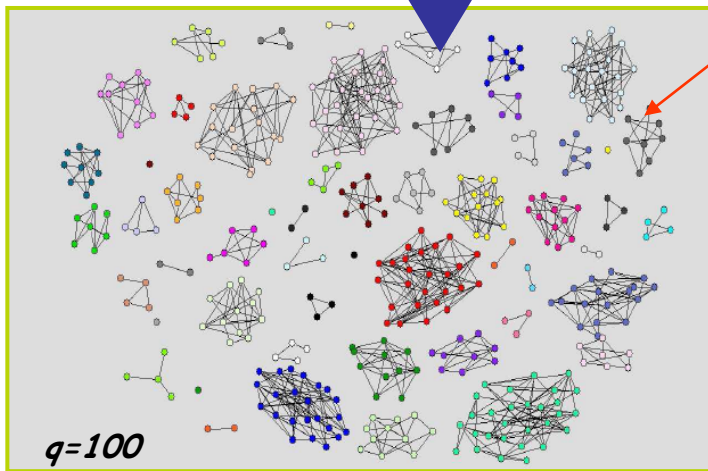


Region I (frozen configuration)



Fragmentation

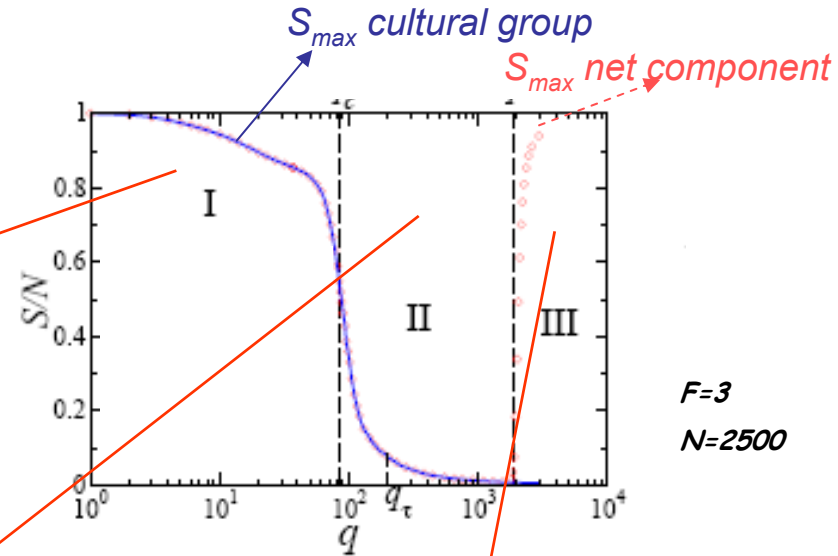
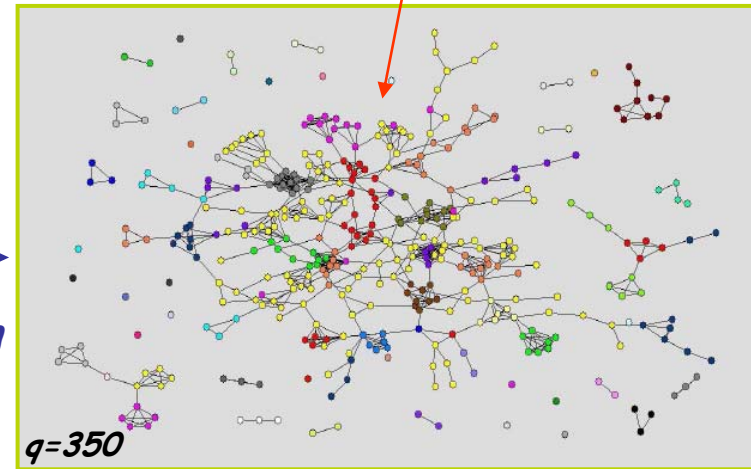
Region II (frozen)



Recombination

$$q^* \cong \frac{NF}{\langle k \rangle}$$

Region III (dynamic frustrated configuration)



* **Question addressed:** Competition between collective social self-organization vs. external mass-media or propaganda message

* **Take home results:**

- 1) Strong messages do not homogenize, but rather produce polarization
- 2) Social interactions can lead to a social consensus different from the external message

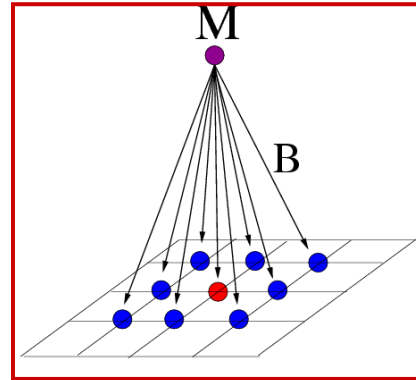
provided there are long range links in the social network of interactions

Mass Media message or field: $M = (\mu_1, \mu_2, \dots, \mu_f, \dots, \mu_F) \quad \mu_f \in \{0, \dots, q-1\}$

External media:
(Big brother)

μ_f given

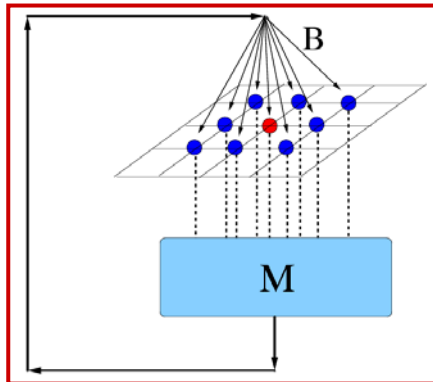
- Uniform for all agents i
- Fixed for all times



Propaganda or advertising

Global media

Broadcast: Feedback of dominant global cultural trend

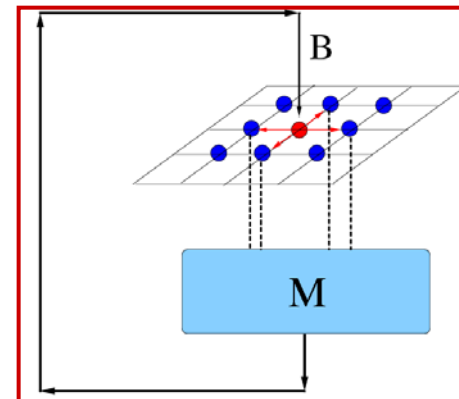


$\mu_f = \sigma_{jf}$ most abundant in system

- Uniform
- Time dependent

Endogenous media:
(4th democratic power)

Narrowcast: Feedback of dominant local cultural trend



$\mu_f = \sigma_{jf}$ most abundant in neighborhood

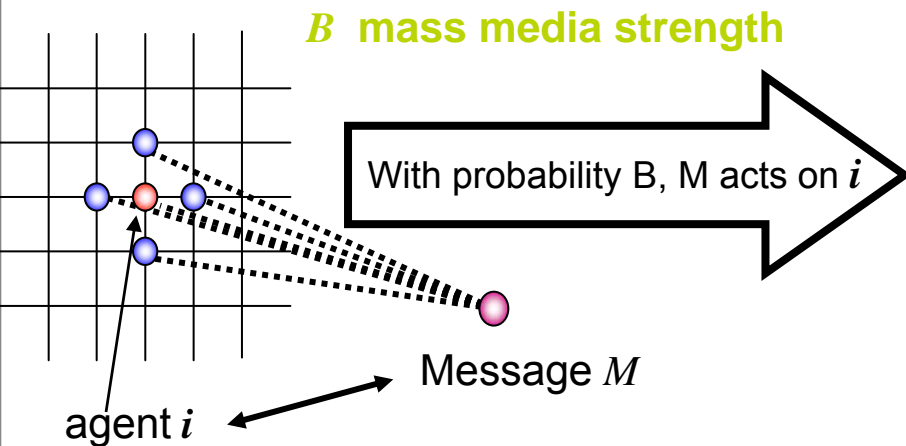
- Non-uniform
- Time dependent

Local media

Agent i : $C_i = (\sigma_{i1}, \sigma_{i2}, \dots, \sigma_{if}, \dots, \sigma_{iF}) \longleftrightarrow$ Mass media: $M = (\mu_1, \mu_2, \dots, \mu_f, \dots, \mu_F)$

Parameter $B \in [0, 1]$: probability that M acts on element i in one time step: “strength” of mass media

1- B : probability to interact with j selected at random among nearest neighbors of i . $\Rightarrow M$ acts as a 5th effective neighbor of i .

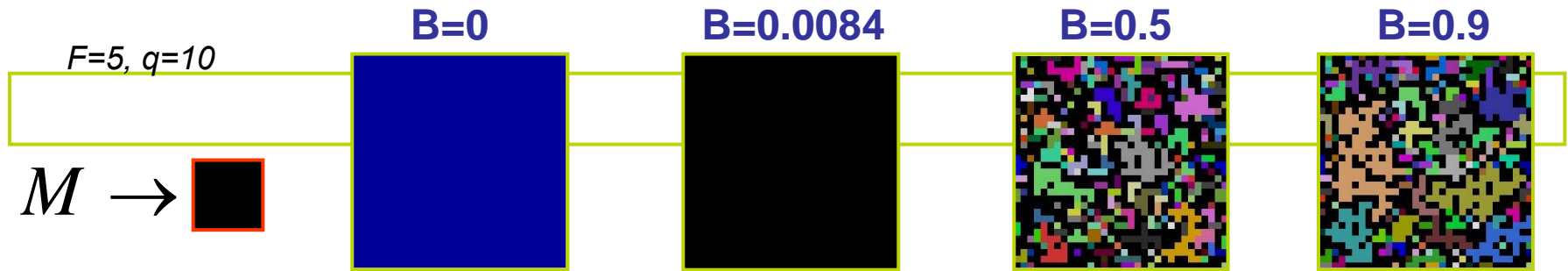


- 1) If M acts on agent i , the probability of interaction p_{iM} is proportional to the cultural overlap between i and M
- 2) Agent-Mass Media interaction results in agent i adopting a cultural feature of M

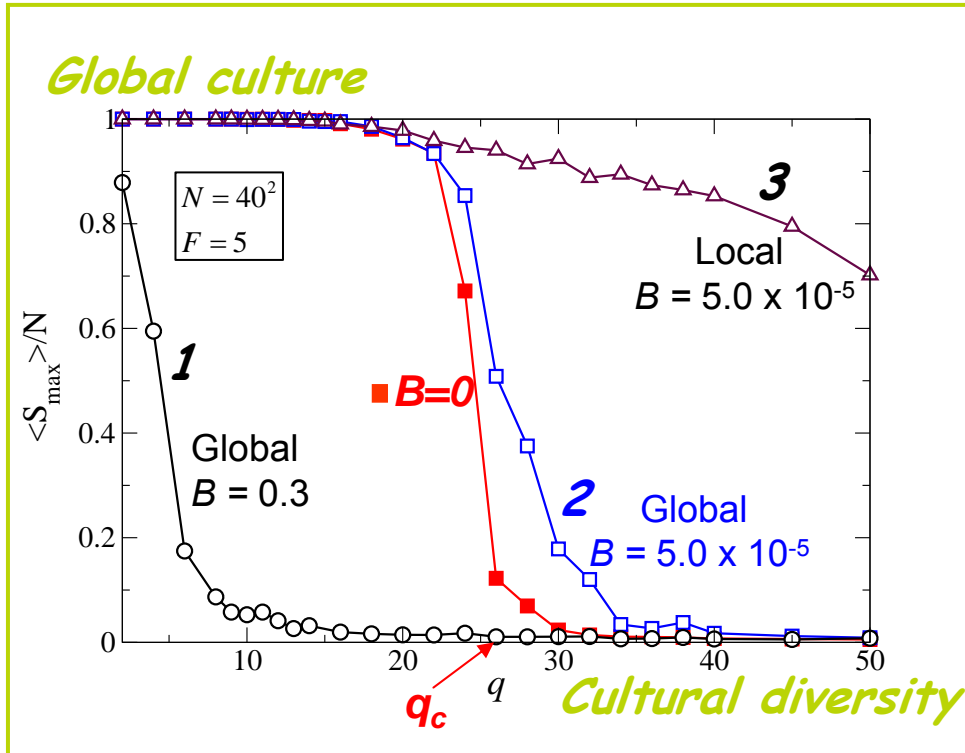
Globalization-polarization transition induced by mass media:

Mass media message produces polarization

Asymptotic states for external mass media



http://ifisc.uib-csic.es/eng/lines/APPLET_Axelrod/Culture.html



1) Polarization caused by strong media ($B > B_c$) →

* Competition of similarity rule applied to agent-agent and agent-media interactions

* Limiting case $B=1$: agent-agent interaction negligible and no agent-media interaction for zero overlap. No mechanism of cultural dissemination at work

2) Cultural homogenization is caused by weak media →

3) Local media (feedback at regional levels) are more efficient in the cultural globalization path.

Mass media is only efficient in producing cultural homogeneity in conditions of weak broadcast of message, so that agent-agent interactions can be still effective in constructing some cultural overlap with the mass media message. Strong media messages do not homogenize because agent-agent interactions become inefficient:

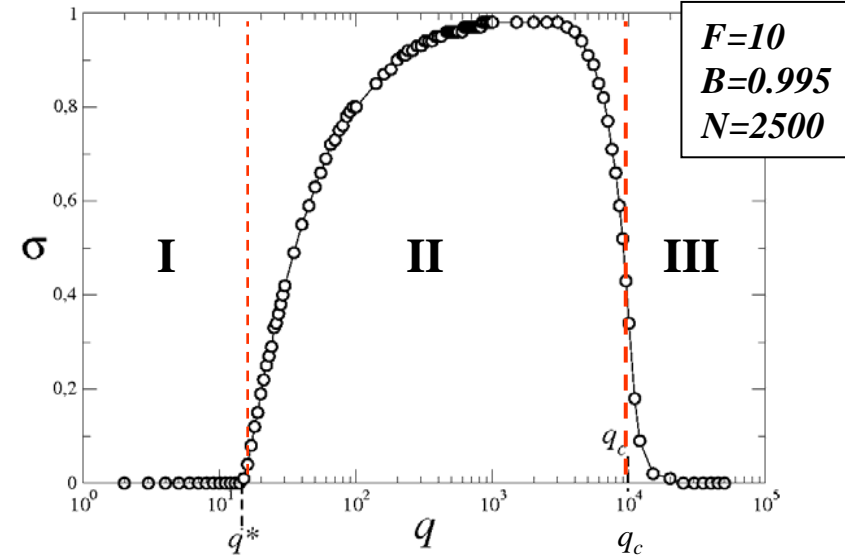
The power of being subtle (and local)

External Media

$$\sigma = \frac{\langle S_{\max} - S_M \rangle}{N}$$

S_{\max} : size of largest domain

S_M : size of domain having state equal to M



Phases:

I: homogeneous, ordered = external field

$$S_{\max} = S_M \neq 0 \quad \text{for} \quad q < q^*(B)$$

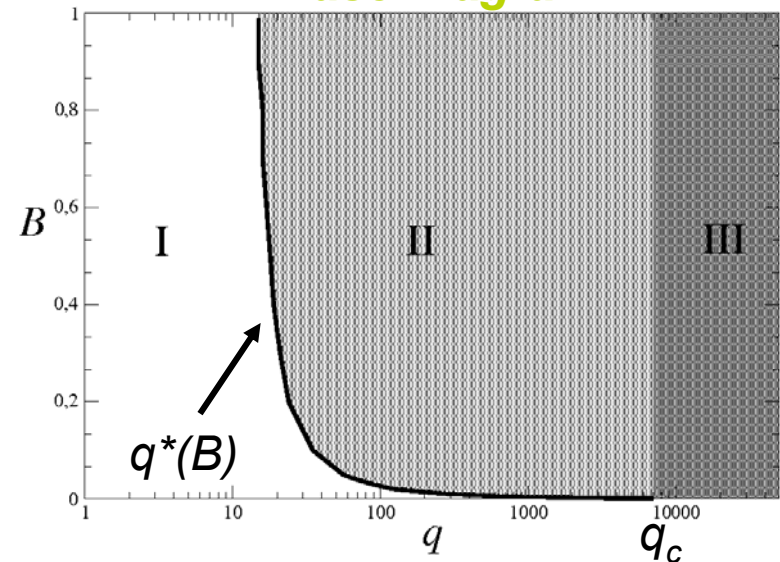
II: alternative ordering state \neq external field

$$S_{\max} > S_M \quad \text{for} \quad q^*(B) < q < q_c$$

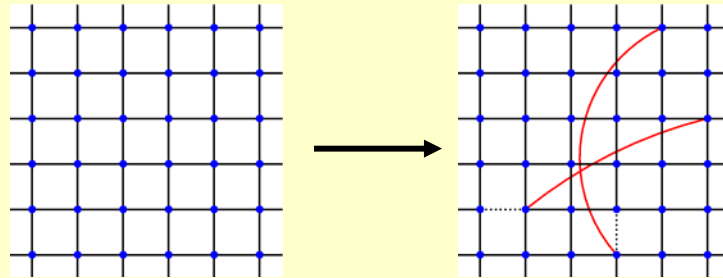
III: disordered

$$S_{\max} \rightarrow 0, S_M \rightarrow 0 \quad \text{for} \quad q > q_c$$

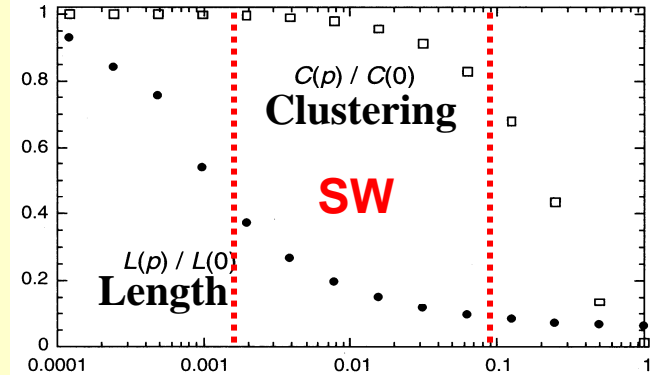
Phase Diagram



Small World Networks

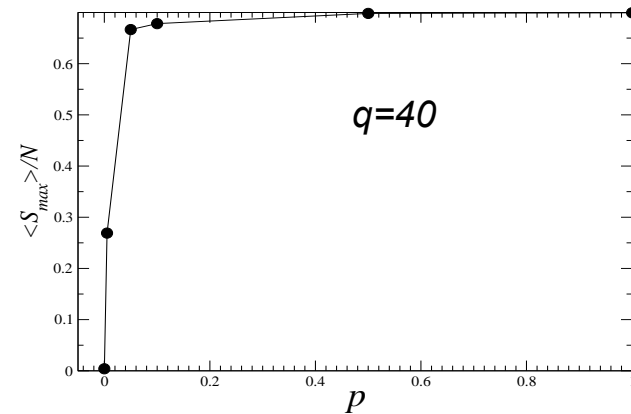
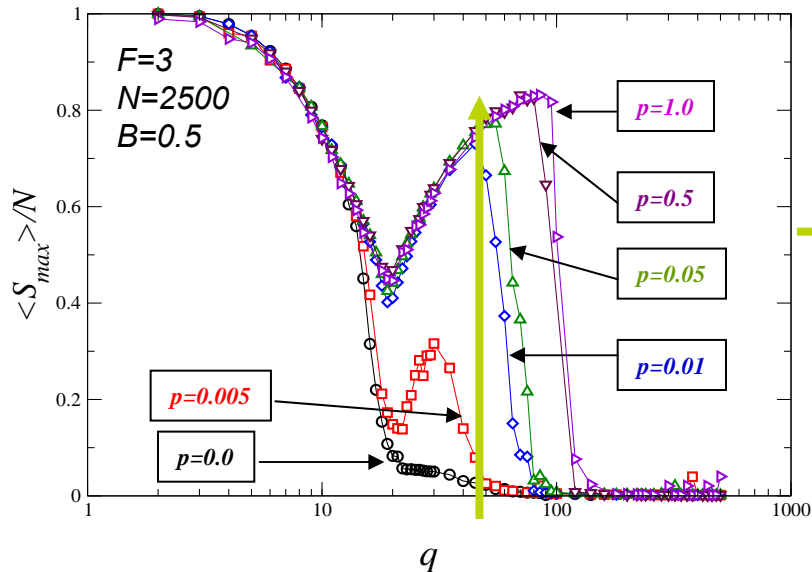


Rewire with prob. p



Regular net.

Random net.



The emergence of a self-organized group opposed to the external message is possible because of the existence of long range social links.

* **Question addressed:** Competition between collective social self-organization vs. external mass-media or propaganda message

* **Take home results:**

- 1) Strong messages do not homogenize, but rather produce polarization
- 2) Social interactions can lead to a social consensus different from the external message

provided there are long range links in the social network of interactions