Braess' Paradox, (In-)Stability and Optimal Design
in Modern Power Grids

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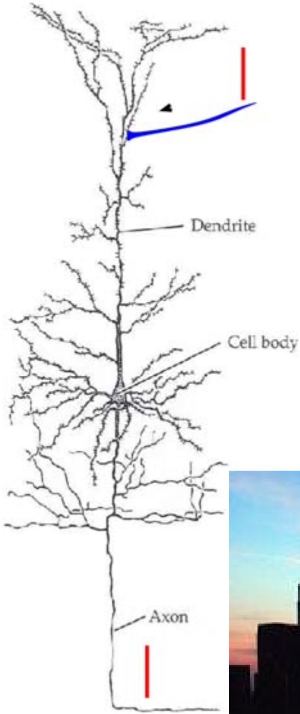
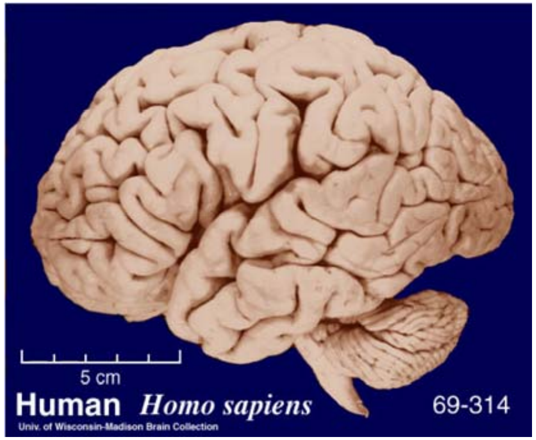
Self-organized Dynamics in Networks of Biology and Physics

<u>Biological Networks</u>

$(10^{-3} - 10^{10}s;\ 10^{-5} - 10^{-1}m)$

- Neurons & neural circuits
- Horizontal gene transfer & evolutionary networks





<u>Networks of physical & artificial units</u>

$(10^{-2} - 10^{10}s;\ 10^{-9} - 10^{6}m)$

- Network growth & disordered media
- Modern power grids (mind the renewables!)
- Autonomous robots & network control

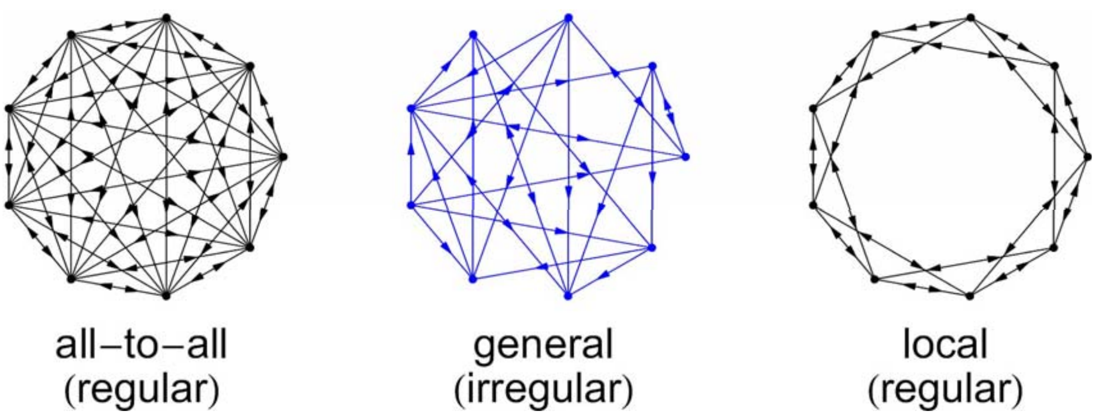
Mathematical Challenges for Theory

Simultaneous occurrence of:

- **Nonlinearity**
- **High dimensionality**
- Complicated Network **Connectivity**
- Interaction **Delays**
- Strong **Heterogeneities**
- **Stochasticity**

<u>common approach:</u>

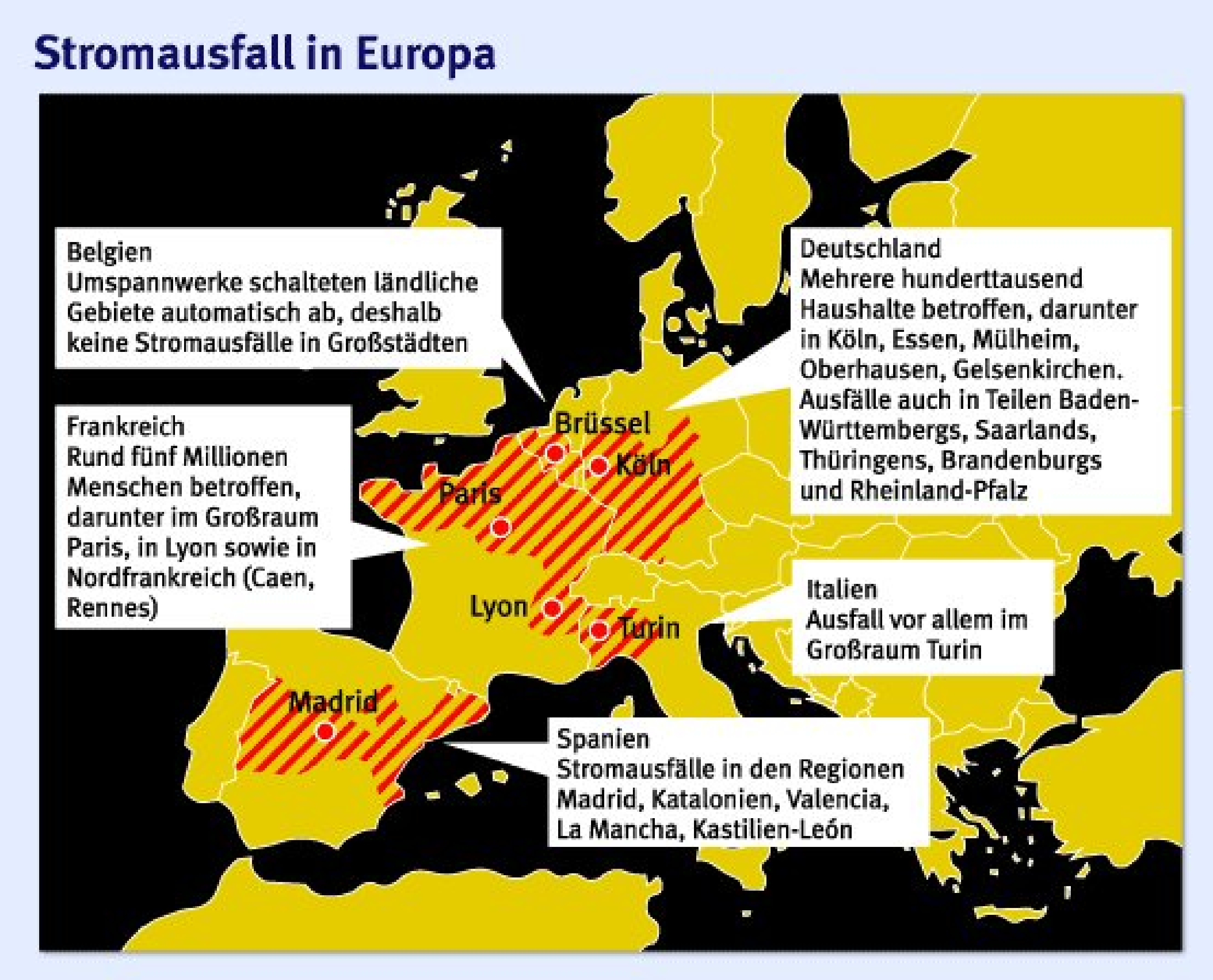
Mean Field Theories, Statistical Description, e.g. averaging over network



Help: **Self-organization & bio-inspired approaches**

Distributed collective grid dynamics ...

Local shut-down ➔ **Nonlocal** impact



Frankf. Allg. Zeitung, Nov 5,2006

... far from fully understood

just **two** single high-power **connections** in northwest Germany **cut**:

→ outages in France, Italy, Austria, some parts of Spain, Portugal, the Netherlands, Belgium and Marocco

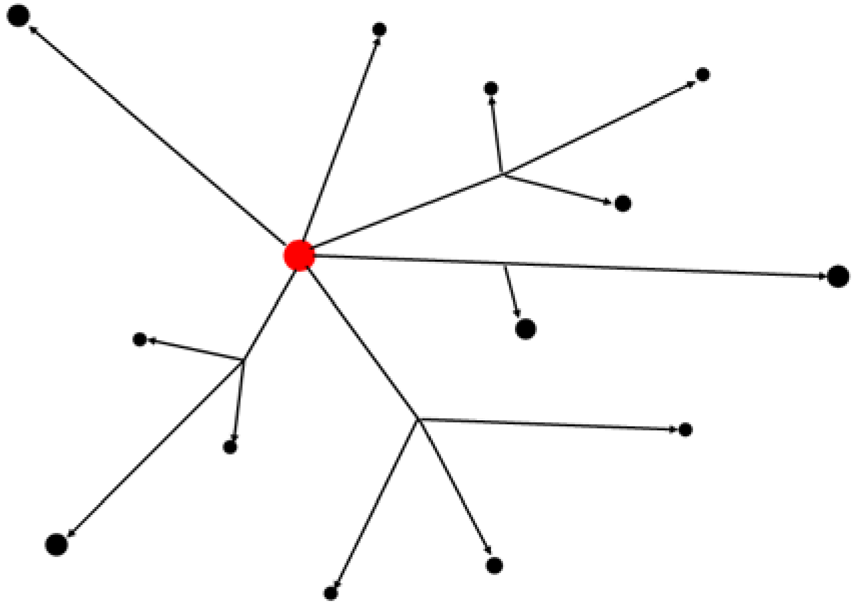
→ **Delayed (30 min)** consequence

>>**"In the past, these operations were often performed with no problems",**
E.O.N. officials declared in great surprise<< (*Softpedia*)

„**We need more interconnections**“
says A.Merlin of RTE, France's power-grid operator *(Bloomberg)*

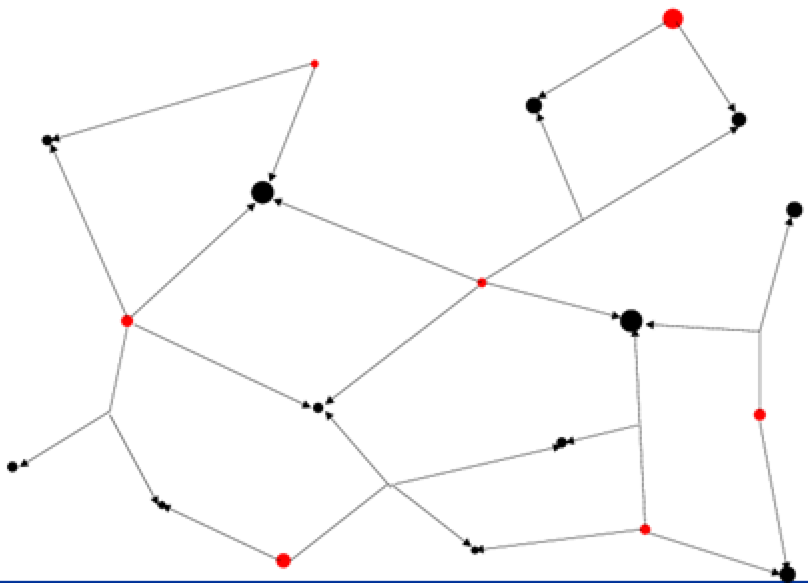
Which factors determine the collective dynamics of power grids?

How does the grid self-organize dynamically?



2000
- largely centralized
- dominated by large sources
- almost central control

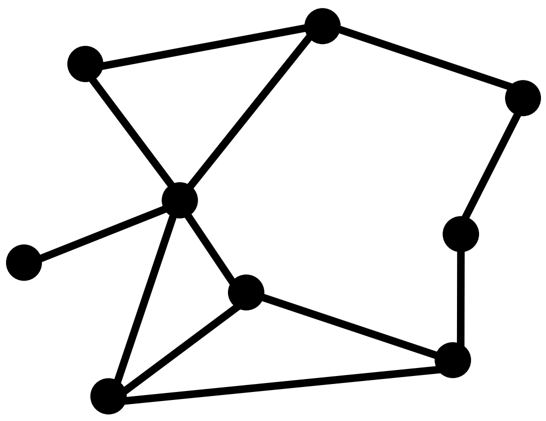
2050
- more distributed
- smaller sources
- less controllable

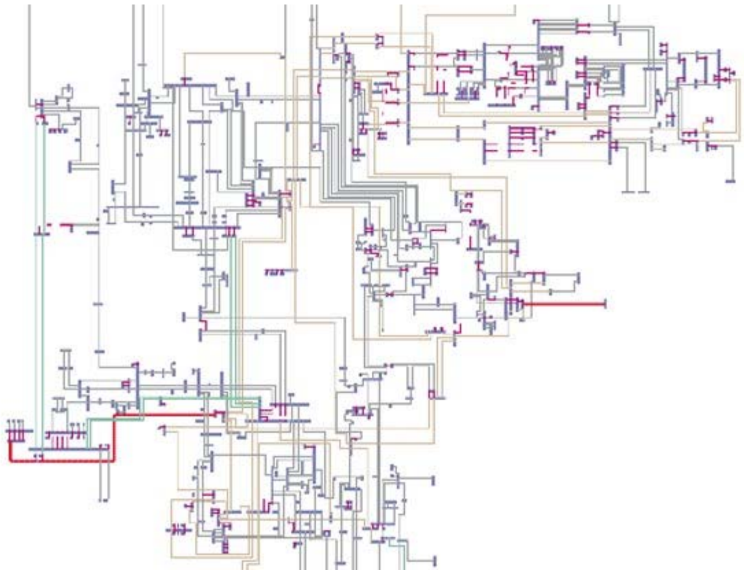


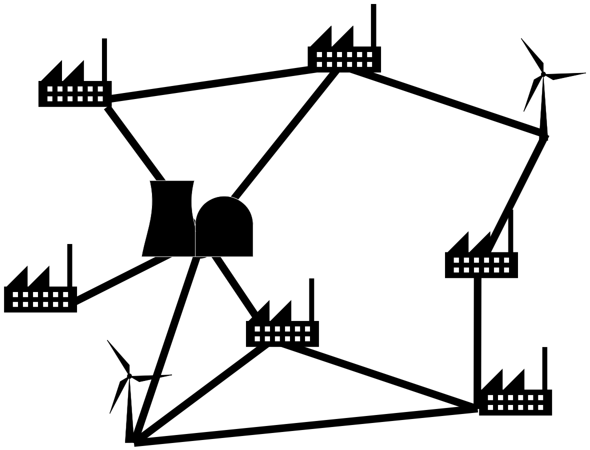
Dynamic oscillator models: between abstract and detailed

<u>Abstract:</u> homogeneous, statistic, quasi-static, large-scale

<u>Detailed</u>: heterogeneous, component-level, dynamic, small-scale

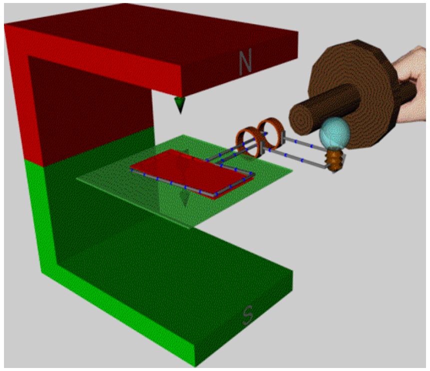


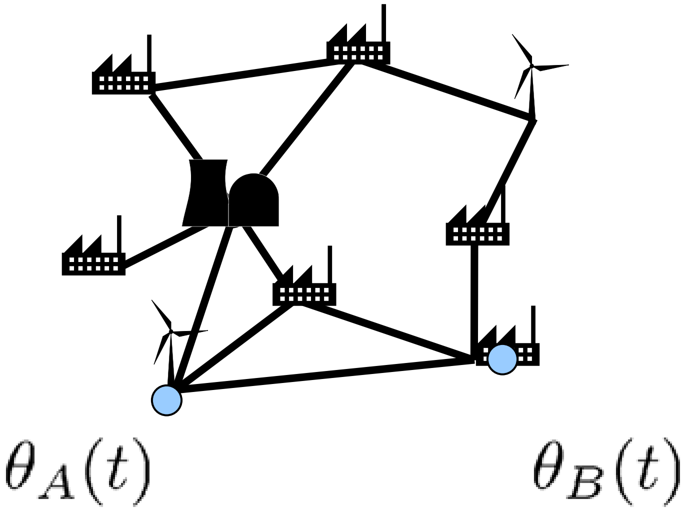


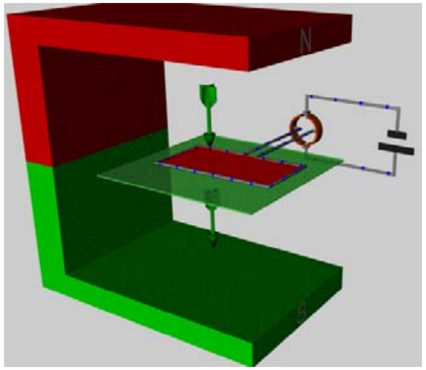


<u>Oscillator model:</u>
- heterogeneous
- coarse-grained
- dynamic
- large-scale

Coarse, dynamic oscillator model of power grids







Generator

Motor

$\theta(t)$	Phase at time t
P	input/output power
K	Transmission capacity

Images: physik3d.de, Wikipedia

Oscillator network as dynamical system

$$\ddot{\theta}_i(t) + \dot{\theta}_i(t) - K \sum_{j \neq i} a_{ij} \sin(\theta_j(t) - \theta_i(t)) = P_i$$

θ_i "mechanical" relative phase of oscillator i

$\dot{\theta}_i$ First derivative (velocity) – damping

$\ddot{\theta}_i$ Second derivative (acceleration) – inertia

K Capacity of each transmission line

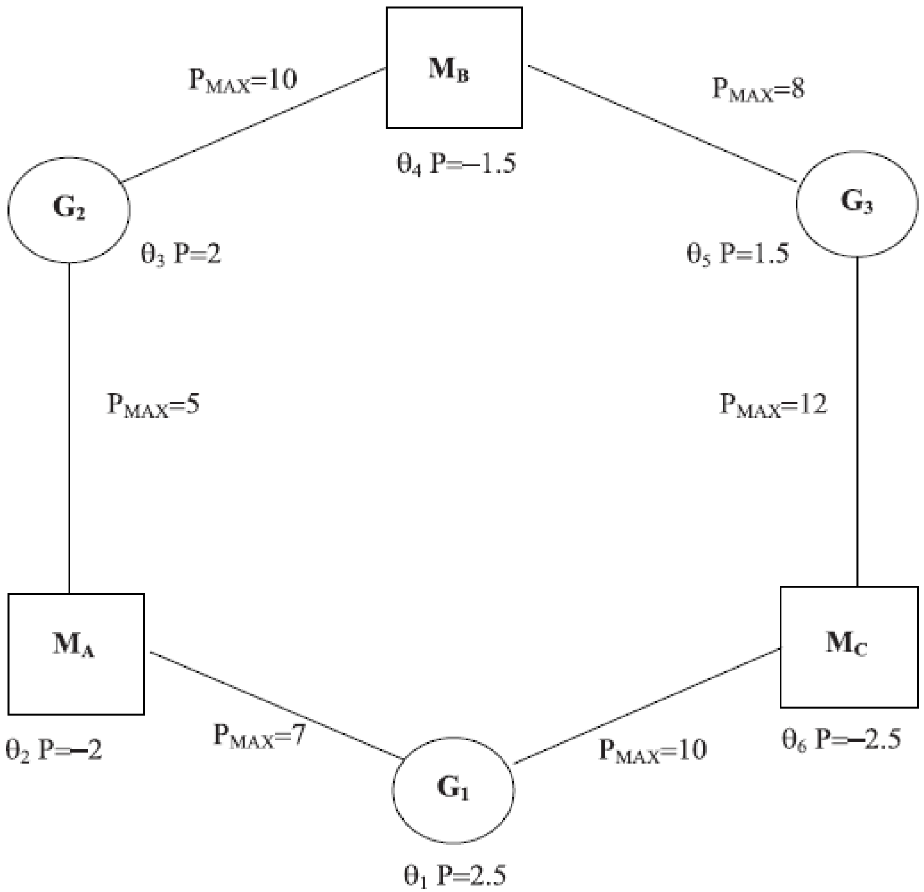
a_{ij} = 1 if there is a line between i and j (=0 otherwise)

P_i Power supplied (+) or consumed (–)

Filatrella et al., *Eur. Phys. J. B*, 61:485 (2008)

Previous studies on this model

Approximation to grid of Zealand (Denmark)



- simple topology
- constant parameters

- simulations of impact of temporary perturbations

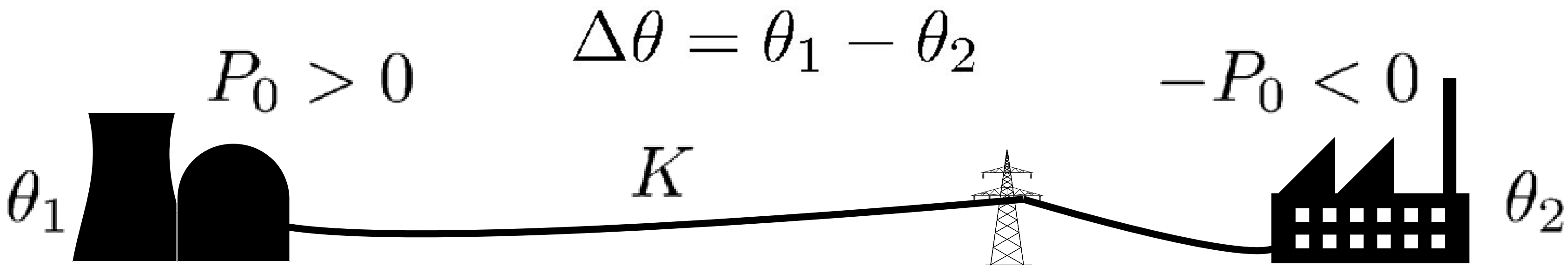
Filatrella et al., *Eur. Phys. J. B*, 61:485 (2008)

Mathematical analysis of stationary dynamics (N=2)

$$\ddot{\theta}_1 + \dot{\theta}_1 - K\sin(-\Delta\theta) = P_0$$

$$-\quad \ddot{\theta}_2 + \dot{\theta}_2 - K\sin(\Delta\theta) = -P_0$$

$$\Delta\ddot{\theta} + \Delta\dot{\theta} + 2K\sin(\Delta\theta) = 2P_0$$



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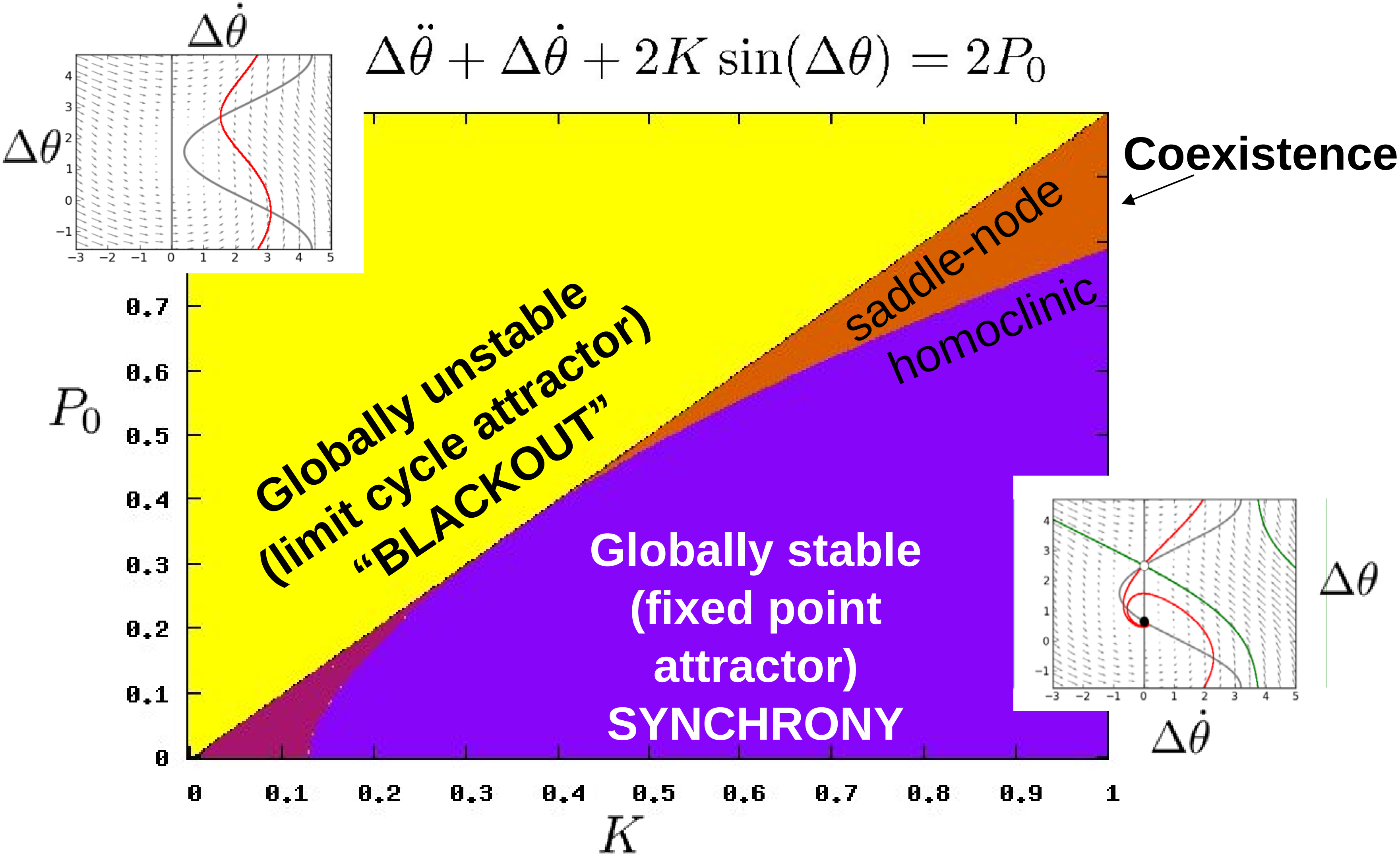
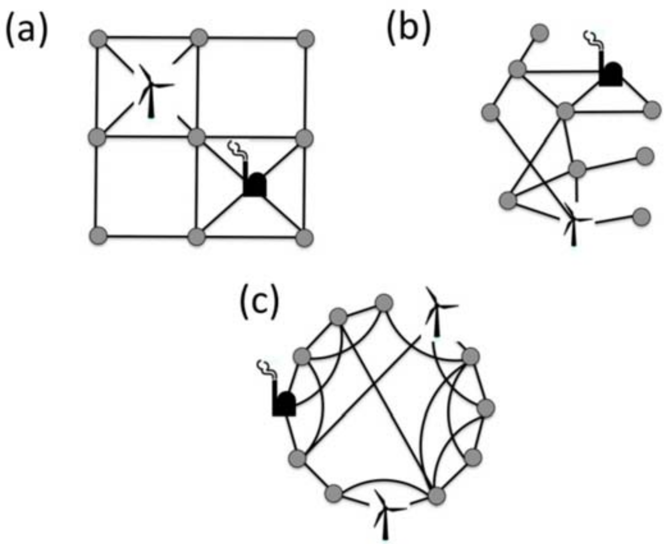
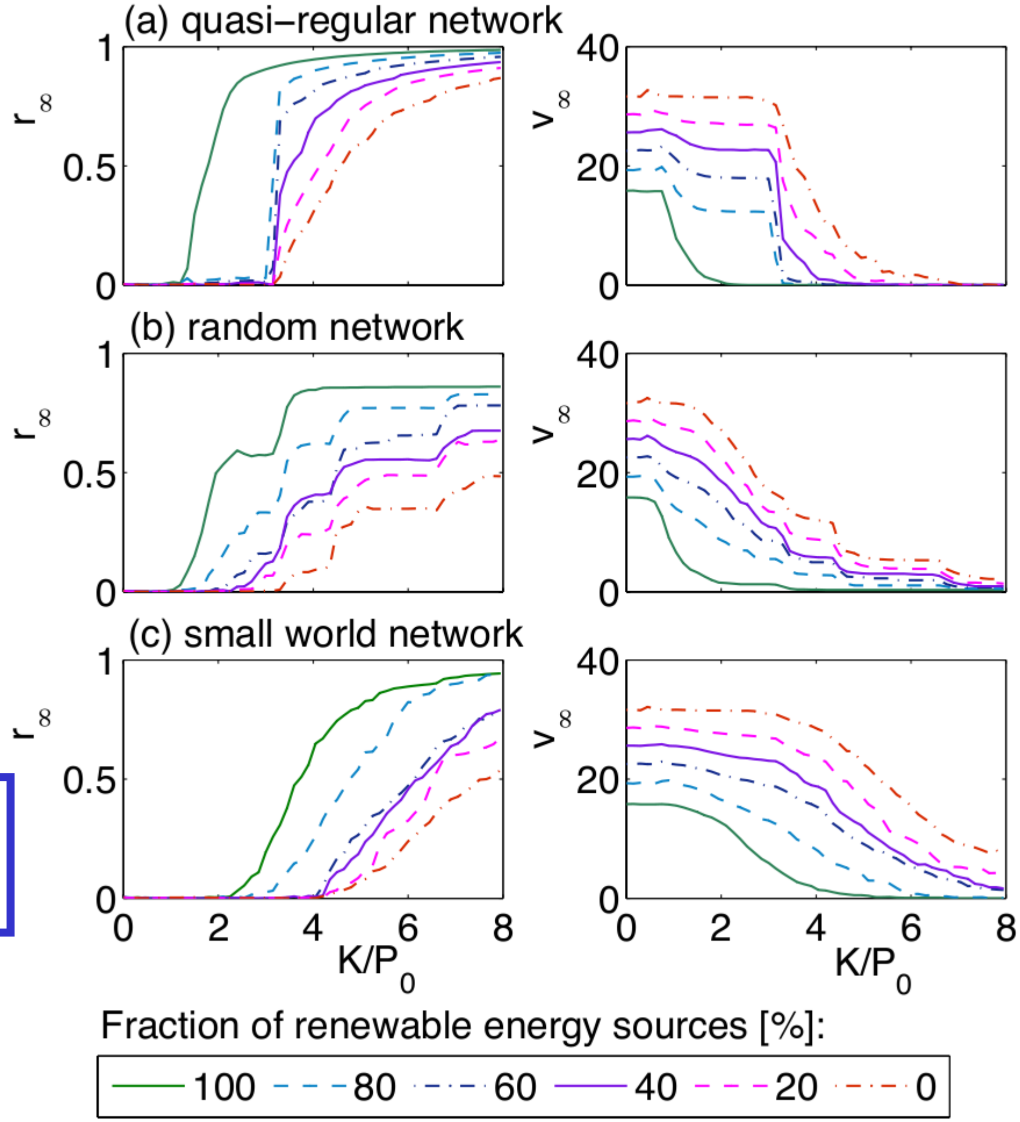


M. Rohden et al., *almost accepted*

Computational analysis of networks of different topologies

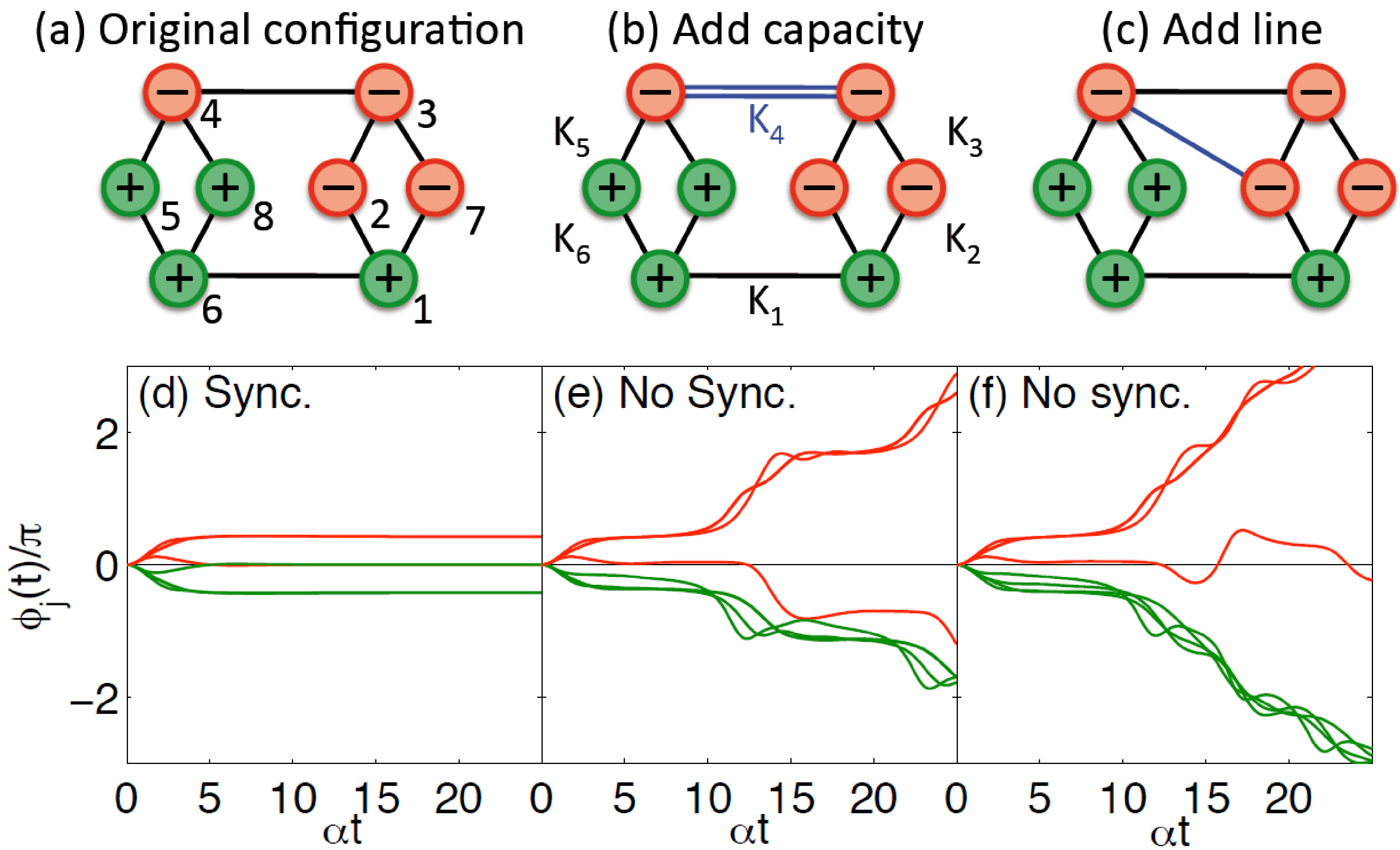




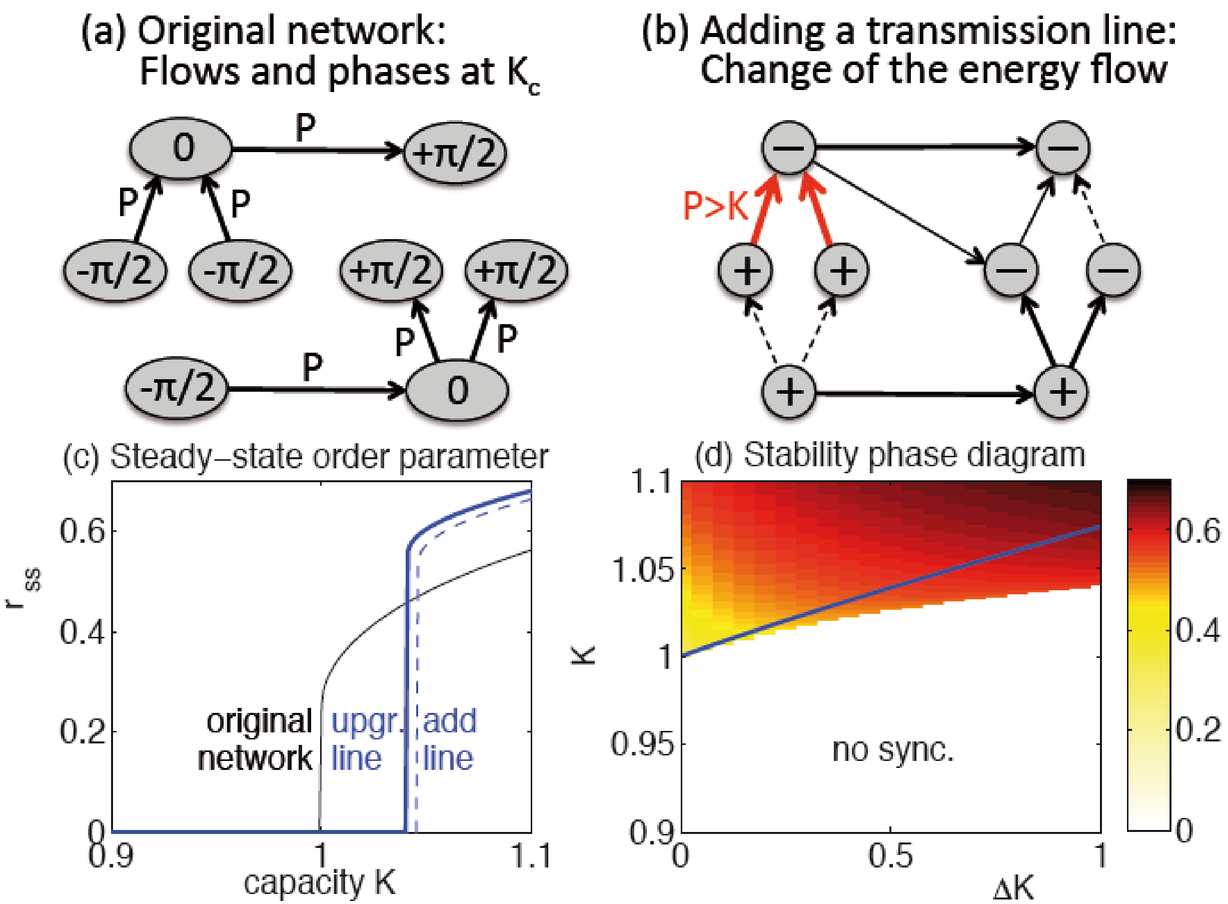
More & smaller sources
➔ larger stationary stability

M. Rohden, et al., *almost accepted, Phys. Rev. Lett. (2012)*

Braess paradox: **adding lines causes failure**



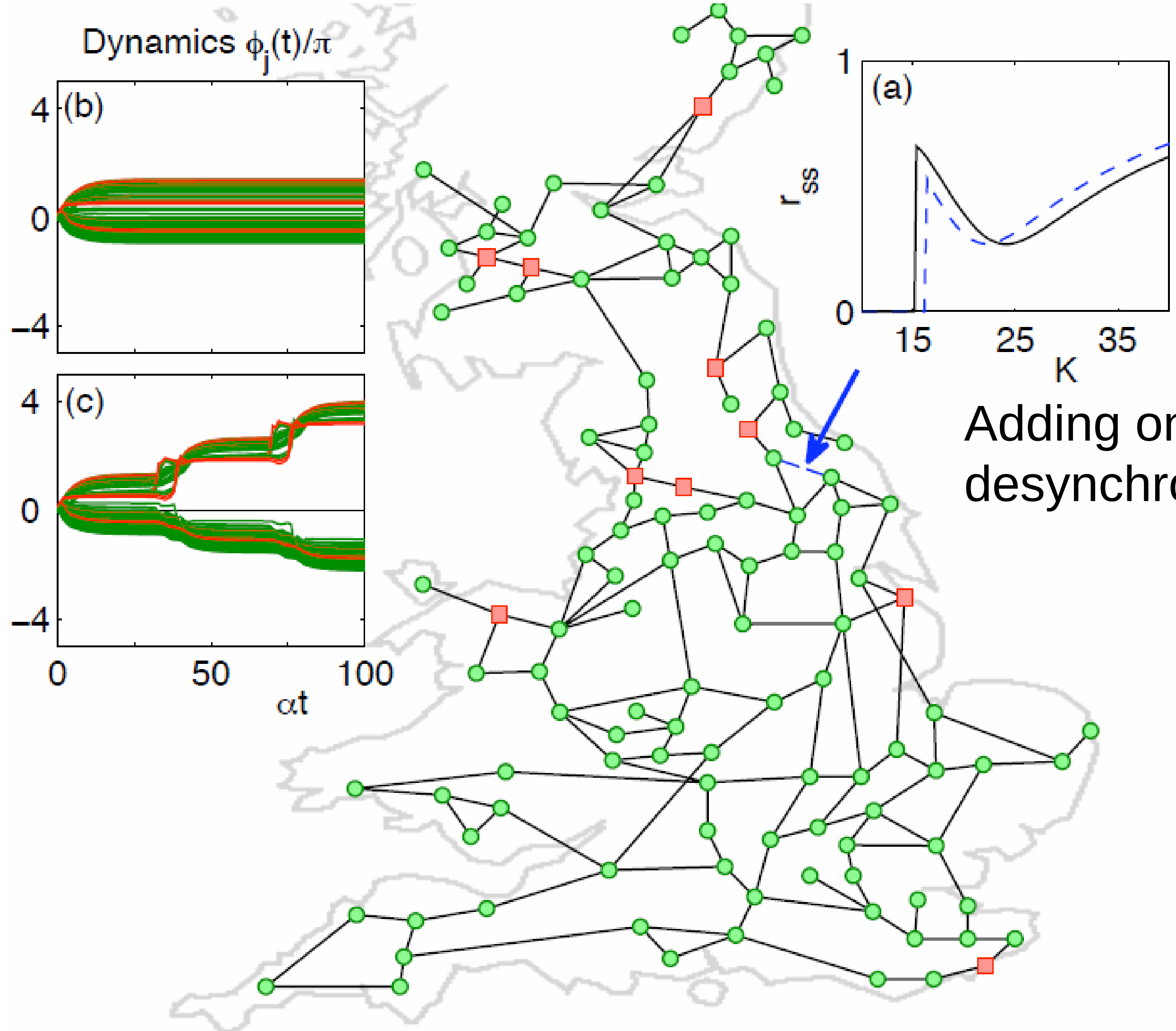
Mechanism of Braess paradox: geometric frustration



Cyclic phase differences need to add up to zero

➔ **Additional capacity „breaks the balance“**

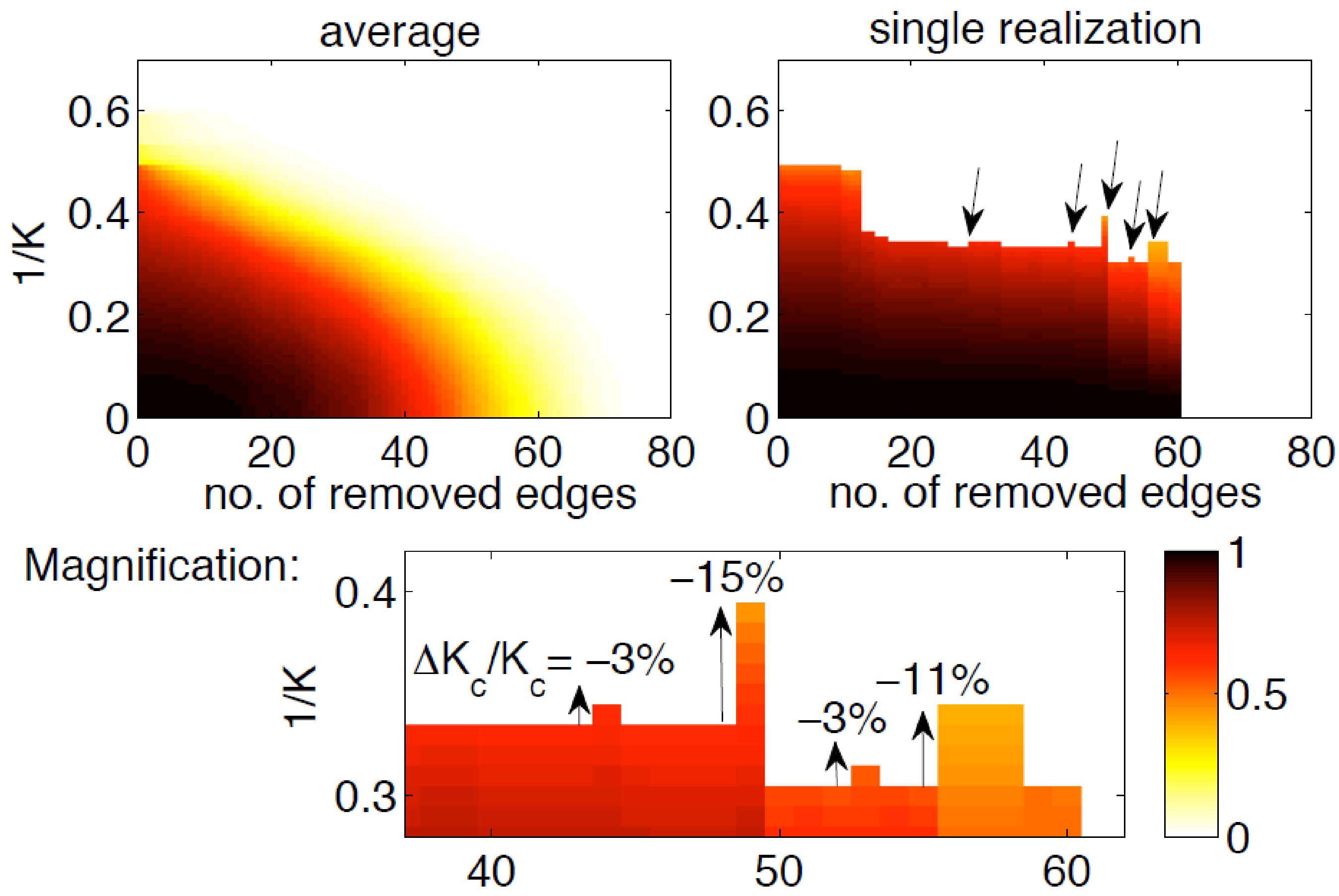
Example of Braess paradox on given topology



Adding one line desynchronizes

Persistent occurrence on model topologies

Small world networks (N=50+50, q=0.1)



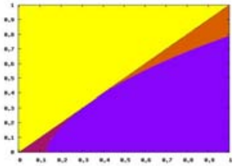
Substantial number of links cause Braess paradox

Dirk Witthaut & MT, under review (2012)

Summary

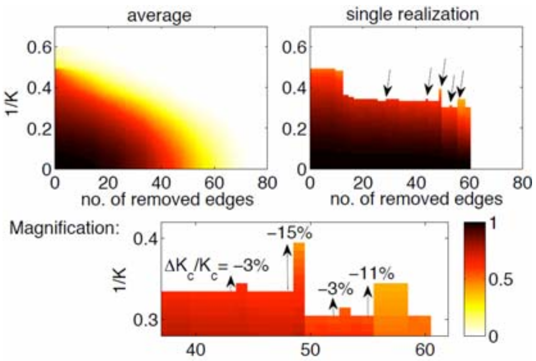
Demonstrated some key capabilities of intermediate oscillator model

• complete understanding of basic determin. system (N=2, bifurcations)



• **distributed sources ➔ larger (stationary) stability**

• Braess paradox (**adding lines may be bad**)



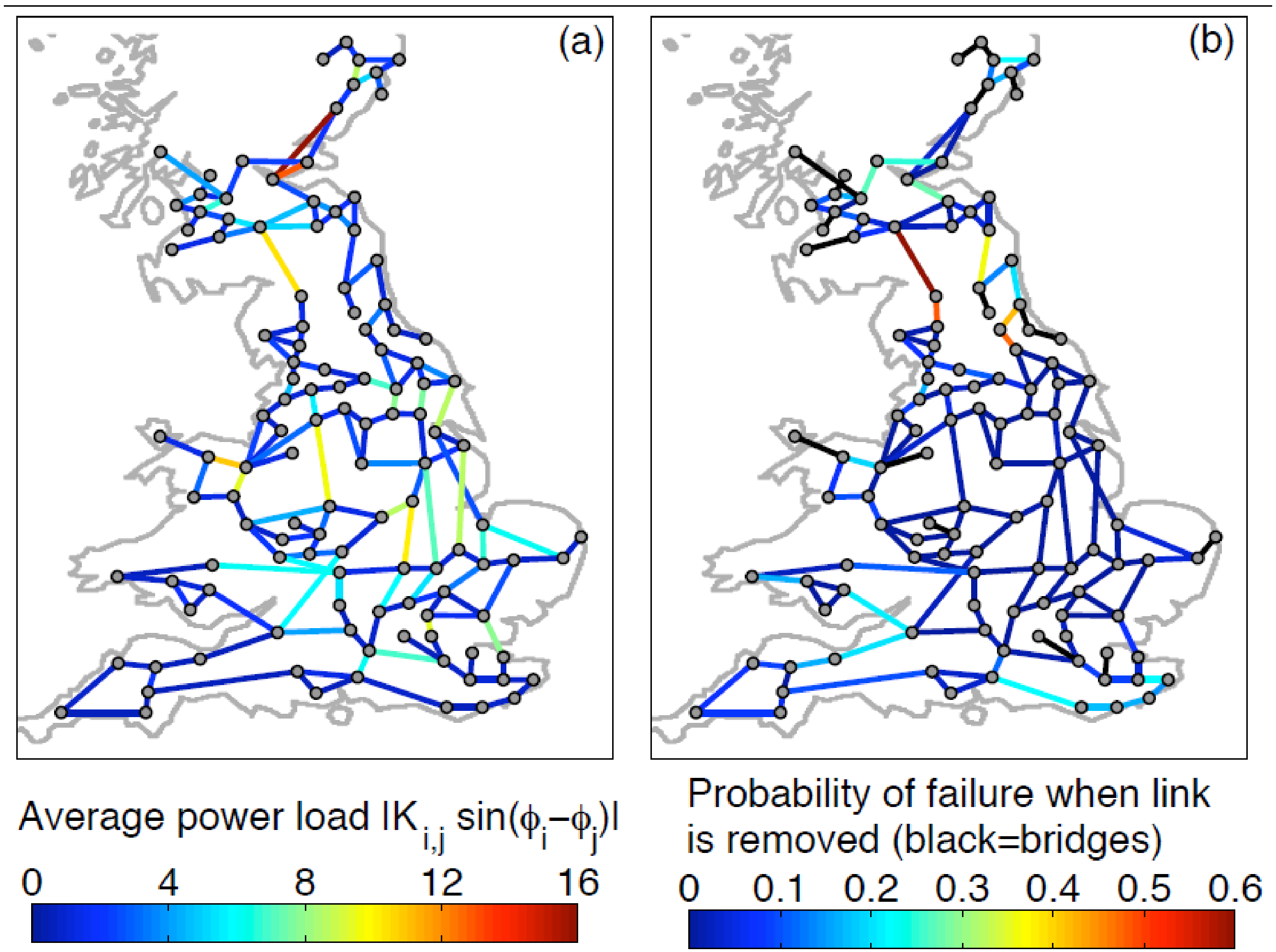
Open Research Questions

Open Questions & work in progress

- response to **instantaneous perturbations** (not shown)?
- Which lines are most critical (not those with highest load) ?
- sensitivity to continuous **fluctuations**
(mathematical and computational analysis) in sources and consumers
- **redistribution** of energy (How to get the power where it is needed)
geographically & time-resolved data (sun, wind, consumer demand)

…

How robust against connection cuts?



Not heavily loaded links, but **least redundant links most critical ?**

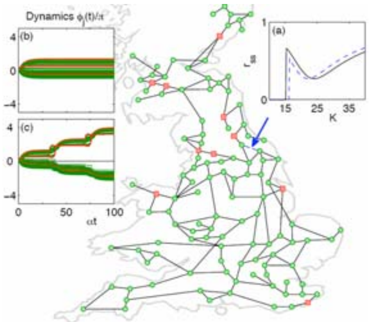
Theoretical Challenges in Network Dynamics:
Nonlinear Dynamics, Inference, Computation & Behavior

- **Network Dynamics and Information Processing**

Phys. Rev. Lett. 89:258701 (2002c);
Phys. Rev. Lett. 92:074101 (2004b);
Chaos 16:015108 (2006);
Phys. Rev. E 78:065201(R) (2008);
Europhys. Lett. 90:48002 (2010);
Chaos, 21:025113 (2011);
Phys. Rev. Lett. 92:074103 (2004a);
Phys. Rev. Lett. 93:074101 (2004c);
Nonlinearity 21:1579 (2008);
Phys. Rev. Lett. 102:068101 (2009);
SIAM J. Appl. Math. 70:2119 (2010);
Phys. Rev. Lett. 107:244101 (2011);

- **Network Inference: Design, Reconstruction and Stability**

Phys. Rev. Lett. 97:188101 (2006);
Europhys. Lett. 76:367 (2006);
Phys. Rev. Lett. 100:048102 (2008);
Frontiers Comp. Neurosci. 5:3 (2010);
***Phys. Rev. Lett.* 108:218701 (2012);**
small world formula
Physica D 224:182 (2006);
Phys. Rev. Lett. 98:224101 (2007);
Frontiers in Comput. Neurosci. 3:13 (2009);
New J. Physics. 13:013004 (2011)
Witthaut & MT, submitted (2012)



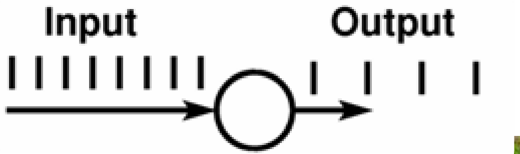
- **Spatio-temporal patterns, control and computation**

Phys. Rev. Lett. 89:154105 (2002a);
Nonlinearity 18:20 (2005);
Neurocomputing 70:2096 (2007);
Frontiers in Neurosci. 3:2 (2009);
Chaos 13:377 (2003);
Nature 436:36 (2005);
Neurosci. Res. 61:S280 (2008);
Discr. Cont. Dyn, Syst. 28:1555 (2010);
Handbook on Biological Networks (Chapter on 'Spike Patterns'), World Scientific (2010);
Bick et al., SIAM J. Appl. Dyn. Syst., accepted, (2012); PLoS Comput. Biol. 8:e1002384 (2012);

Theoretical Challenges in Network Dynamics: **Nonlinear Dynamics, Inference, Computation & Behavior**

- **Adaptation** and **autonomous robots** via nonlinear dynamics

  Nature Phys. 6:224 (2010); *J. Phys. A: Math. Theor.* 42:345103 (2009);
  Phys. Rev. Lett., under revision (Kielblock et al., 2012)
  ***Phys. Rev. Lett.,* 109, 018701 (2012),**
  new paradigm: heteroclinic computing



- **Intelligent coordination and new computational devices**

  Phys. Rev. Lett. 88:245501 (2002b); *Cornell Rep.* 1813:1352 (2007);
  New J. Phys. 11:023001 (2009); *J. Phys. A: Math. Theor.,* 43:175002 (2010);
  Nature Phys., 7:265 (2011); New J. Phys., under review (2012)

Thanks to ...

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YOU all for your attention !

Questions & Comments Welcome!