Temporal Correlation of Interference: Cases with Correlated Traffic

Udo Schilcher 1, Günther Brandner 1, and Christian Bettstetter 1,2
1 University of Klagenfurt, Institute of Networked and Embedded Systems, Mobile Systems Group, Austria
2 Lakeside Labs GmbH, Klagenfurt, Austria

Abstract—We analyze the dynamics of interference in a wireless network under a correlated traffic model: A given node increases or decreases its transmission probability if it has transmitted in the preceding time slot. This traffic model is of importance since it resembles part of the behavior of many communication techniques as, e.g., retransmission protocols or cooperative relaying. Results show that the traffic model has high impact on temporal correlation of interference, which has to be considered when designing such techniques.

Index Terms—Interference, correlation, retransmission protocols, wireless networks, Rayleigh fading, slotted ALOHA.

I. INTRODUCTION AND MOTIVATION

The quality of radio links in wireless networks changes over time. This behavior called fading is caused by movement of the nodes and objects reflecting radio waves and by transmitters causing interference. Packets can be lost in periods in which the channel is ‘bad’. Many communication techniques and protocols counteract these channel conditions, e.g., retransmission protocols [1], channel coding and interleaving [2], cooperative relaying [3], and opportunistic scheduling [4]. Good knowledge of the channel dynamics is essential when designing such techniques. Although fading dynamics has received a lot of attention in the past, interference dynamics has not. Only recently the research community gained some interest in the dynamics of interference [5]–[9].

A first step toward a better understanding of the temporal behavior of interference is to identify important causes for interference correlation and to analyze three extreme cases: the cause is not considered, the cause changes independently from time to time, and the cause does not change at all [7]. The next step is to generalize these results to more realistic cases in between these extreme setups. First results on this are presented in [8] and [9], where the dynamics of the node locations is enriched by different mobility models. This setup resembles the case where node locations neither stay constant over time nor change independently in each time step.

In the paper at hand we apply a similar principle to the traffic as a cause of interference correlation. While previous work analyzed extreme cases, where either senders are independently chosen or retransmit with probability one, this paper analyzes a case in between. We apply a traffic model that increases or decreases the transmission probability dependent on whether a given node sent in the preceding time period or not. Based on these assumptions we derive closed-form expressions for the correlation of interference.

II. MODELING ASSUMPTIONS

The correlation of two random variables \(X\) and \(Y\) is measured in terms of Pearson’s correlation coefficient

\[
\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\sqrt{\text{var}(Y)}}. \quad (1)
\]

The terms \(\text{var}\) and \(\text{cov}\) denote the variance and covariance, respectively.

A. Node Locations

The nodes of the network are located at the points of a Poisson Point Process (PPP) \(\mathcal{N}\) with density \(\lambda\). For simple notation let \(\mathcal{N}\) denote both the PPP and the set of nodes. Let \(\|x\|\) denote the distance of a node \(x \in \mathcal{N}\) to the origin \((0,0)\). The locations of the nodes are fixed, i.e., no mobility is applied. Fixed locations can also introduce a correlation of interference [5], [6]. We are excluding this correlation from our analysis by assuming that the node locations are a given condition for the calculations.

B. Channel

All nodes send with the same transmission power \(\kappa\) in \(W\). The wireless channel is modeled via path loss and Rayleigh block fading. All nodes access the wireless channel applying Slotted ALOHA, where time is divided into discrete slots indexed by \(\tau \in \mathbb{N}\). In our correlation analysis we compare two arbitrarily chosen consecutive time slots indexed by \(\tau = t - 1\) and \(t\). Let \(\mathcal{S}_\tau \subseteq \mathcal{N}\) denote the subset of nodes transmitting in slot \(\tau\).

By interference we denote the overall power received by an antenna at a given point in space from all transmitting nodes (except the signal the node is intended to receive). Without loss of generality we limit our analysis to an antenna at the origin \((0,0)\) of the plane \(\mathbb{R}^2\). The interference at \((0,0)\) in slot \(\tau\) can be expressed by [10]

\[
I_\tau = \sum_{x \in \mathcal{S}_\tau} \kappa l(\|x\|) h_x^2. \quad (2)
\]

In this equation, \(l(\|x\|)\) denotes the path loss at distance \(\|x\|\). It is possible to apply any non-singular (i.e. \(\lim_{x \to 0} l(\|x\|) < \infty\)
path loss model; we assume \( l(\|x\|) = \min(1, \|x\|^{-\alpha}) \) with a path loss exponent \( \alpha > 2 \). The exponentially distributed random term \( h_x^\rho \) models Rayleigh fading, i.e., it denotes the current channel state for node \( x \). Fading is assumed to be independent for each sender: \( h_x^\rho \overset{i.i.d.}{\sim} \text{Exp}(1) \) for all \( x \).

A block fading model is applied, in which the channel is held constant for a certain time \( c \) and is then newly chosen independently from previous states (c.f. [11], [12]). For the time period \( c \) we distinguish three different cases:

0) We assume a constant channel over time, i.e. no fading is present, \( h_x^\rho \equiv 1 \).

1) The channel is changing at the beginning of each time slot, which means \( c = 1 \). Hence, we have independent fading for each slot.

2) The channel changes after \( c \geq 2 \) slots. This slow fading channel increases the correlation of interference.

C. Traffic

Traffic is modeled in the following way: In each slot every node sends a message with a certain probability. The probability of slot \( t \) depends on the previous slot \( t - 1 \): if a node sent a packet in slot \( t - 1 \), it again transmits in slot \( t \) with probability \( p_1 \) (the so-called retransmission probability). Otherwise, it transmits in slot \( t \) with probability \( p_0 \). If \( p_1 = p_0 \) slot \( t - 1 \) has no influence on slot \( t \). If \( p_1 < p_0 \), a node reduces its transmission probability if it sent one slot before. This could model a protocol for which a node waits for an acknowledgment before sending again. Case \( p_1 > p_0 \) could model a retransmission protocol that immediately detects a lost packet and retransmits it in the next slot [1].

D. Case Notation

Using the notation introduced in [7], the three cases analyzed in this paper are denoted by \((0, 0, 3), (0, 1, 3)\), and \((0, 2, 3)\). The meaning of the numbers is the following: The first number (here always 0) indicates that the node locations are not taken into account for calculating the correlation. The second number denotes one of the three cases explained in II-B for the channel. The third number (here always 3) finally denotes that the traffic model described above is applied.

III. DERIVATION OF CORRELATION COEFFICIENT

Let \( S_{11}, S_{10}, \) and \( S_{01} \) denote the set of nodes that send in both slots \( t - 1 \) and \( t \), that send in \( t - 1 \) but not in \( t \), and that send in \( t \) but not in \( t - 1 \), respectively. Further, let \( S \) denote the fractions of nodes within these sets and \( I \) denote the interference caused by the nodes in these sets. The major task is to derive the fractions of nodes that are within these sets. Table I summarizes the results of this paper.

A. Fraction of Nodes in \( S_{11}, S_{10}, \) and \( S_{01} \)

We start by deriving the expected fraction of senders \( E[S_i] \) in a slot \( t \). \( E[S_i] \) mainly depends on the expected fraction of senders \( E[S_{t-1}] \). More specifically

\[
E[S_t] = E[S_{t-1}] p_1 + (1 - E[S_{t-1}]) p_0 .
\]

Since we are interested in the long run, we calculate the fraction of nodes transmitting in the limit for \( t \to \infty \). Therefore, we calculate the fixed point \( S_{FP} \) of (3) by solving \( E[S_t] = E[S_{t}] p_1 + (1 - E[S_t]) p_0 \), which yields

\[
S_{FP} = \frac{p_0}{1 - p_1 + p_0}.
\]

Next, we calculate the fractions of nodes within the sets. \( S_{11} \) contains all nodes sending in \( t - 1 \) that also send in \( t \), which gives

\[
E[S_{11}] = S_{FP} p_1 = \frac{p_0 p_1}{1 - p_1 + p_0}.
\]

The set \( S_{10} \) contains all nodes sending in \( t - 1 \) that do not send in \( t \), which gives \( E[S_{10}] = S_{FP} (1 - p_1) \). Similarly, the set \( S_{01} \) contains all nodes that do not send in \( t - 1 \) but send in \( t \), which gives \( E[S_{01}] = (1 - S_{FP}) p_0 \). Both equations yield the same result, i.e.

\[
E[S_{10}] = E[S_{01}] = \frac{(1 - p_1) p_0}{1 - p_1 + p_0}.
\]

B. Interference Correlation for Case \((0, 0, 3)\)

Let \( N_1 \) and \( N_2 \) denote two randomly chosen disjoint subsets of \( N \). If many strong interferers are within \( N_1 \), they are not within \( N_2 \), because the sets are disjoint. This property introduces a negative correlation of the interference caused by nodes within \( N_1 \) and \( N_2 \). Theorem 1 in [7] enables us to calculate the correlation of the interference caused by these two sets based on the fraction of nodes they contain. Applying this theorem to the sets \( S_{11}, S_{10}, \) and \( S_{01} \) yields

\[
\rho(I_{11}, I_{10}) = -\sqrt{\frac{p_0^2 p_1}{1 + p_0 + p_1 (p_0^2 - 1)}},
\]

\[
\rho(I_{10}, I_{01}) = -\frac{p_0 (1 - p_1)}{1 + p_1 (p_0^2 - 1)}.\]

Next, we derive the correlation coefficient \( \rho(0, 0, 3) \) based on these results. Let \( \gamma_x(S_{10}) \) denote the indicator variable that node \( x \in S_{10} \). It is Bernoulli distributed with variance

\[
\text{var}(\gamma_x(S_{10})) = E[S_{10}] (1 - E[S_{10}]).
\]

The indicator variables \( \gamma_x(S_{01}), \gamma_x(S_{11}) \), and \( \gamma_x(S) \) are defined in a similar way.

TABLE I: Correlation of Interference: Summary of Results

<table>
<thead>
<tr>
<th>Locations</th>
<th>Channel</th>
<th>Traffic</th>
<th>Interference correlation</th>
<th>Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>( j )</td>
<td>( k )</td>
<td>( \rho )</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>( \frac{p_1 - p_0}{(1-p_1)(p_0 + p_1)} )</td>
<td>(15)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>( \frac{p_0 (1+p_0) + p_1 (1+p_1) - 2p_1^2}{2p_1 (p_1 (1+2p_1) - 2p_2)} )</td>
<td>(21)</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>3</td>
<td>( \frac{p_1 - p_0}{(2-p_1 + p_0)(p_1 - p_0^2 - 1)} )</td>
<td>(26)</td>
</tr>
</tbody>
</table>
The expected variance of interference caused by nodes $S_{10}$ can be derived by

$$
E[\text{var} (I_{10} | \mathcal{N})] = E \left[ \text{var} \left( \sum_{x \in \mathcal{N}} \kappa_l(||x||) \gamma_x(S_{10}) | \mathcal{N} \right) \right]
$$

\begin{align*}
&= \kappa^2 \cdot E \left[ \sum_{x \in \mathcal{N}} I^2(||x||) \cdot \text{var}(\gamma_x(S_{10})) \right] \\
&= \kappa^2 \cdot \lambda \int_{\mathbb{R}^2} I^2(||x||) \, dx \cdot \frac{(1 - p_1)(1 + p_1 p_0 - p_1) p_0}{(1 - p_1 + p_0)^2} \cdot \alpha^2 \cdot \frac{\alpha \pi}{\alpha - 1} \cdot \frac{p_1(1 - p_1) p_0(1 + p_0)}{(1 - p_1 + p_0)^2}.
\end{align*}

(10)

The second equation holds since $\kappa$ and $I^2(||x||)$ are constants regarding the variance operator. The third equation holds due to Campbell’s theorem, and (6) substituted into (9). Following the same steps except applying (5) instead of (6) yields

$$
E[\text{var} (I_{11} | \mathcal{N})] \equiv \kappa^2 \cdot \lambda \frac{\alpha \pi}{\alpha - 1} \cdot \frac{p_1(1 - p_1) p_0(1 + p_0)}{(1 - p_1 + p_0)^2}.
$$

In the same way, we can derive the variance of interference $I$, which gives

$$
E[\text{var} (I | \mathcal{N})] \equiv \kappa^2 \cdot \lambda \frac{\alpha \pi}{\alpha - 1} \cdot \frac{p_1(1 - p_1)}{(1 - p_1 + p_0)^2}.
$$

(12)

The covariances in this expression can be computed by rearranging (1) to

$$
E[\text{cov}(I_{11} + I_{10}, I_{11} + I_{01} | \mathcal{N})] = E[\text{cov}(I_{11}, I_{11} | \mathcal{N})] + E[\text{cov}(I_{11}, I_{01} | \mathcal{N})] \quad \text{and similar for } E[\text{cov}(I_{10}, I_{01} | \mathcal{N})].
$$

(13)

By substituting all results derived above into these last two equations and by dividing (13) by (12) we can calculate the correlation of interference for two consecutive time slots in case $(0, 0, 3)$ by

$$
\rho(0, 0, 3) = \frac{(p_1 - 1)(p_1 - p_0)}{p_1 - 1} \cdot \frac{1}{p_1 - p_0}.
$$

(15)

Next, we distinguish between increased and decreased retransmission probabilities. First, we consider an increased probability $p_1 > p_0$. For easier comparison of different values of $p_0$ and $p_1$, let $p^+$ denote the linear increase of the transmission probability after a node has sent, i.e., $p_1 = (1 - p^+) p_0 + p^+$. We have $p_1 = p_0$ for $p^+ = 0$ and $p_1 = 1$ for $p^+ = 1$. Substituting this equation into (15) gives

$$
\rho(0, 0, 3) = p^+(1 - p_0)
$$

(16)

for $0 \leq p^+ < 1$. Note that $p^+ = 1$ implies $p_1 = 1$ and further that the denominator of (15) equals 0. Calculating the limit yields

$$
\lim_{p^+ \to 1} \rho(0, 0, 3) = \frac{(1 - p_0)^2}{1 - p_0} \cdot 1 - p_0,
$$

(17)

which corresponds to (16). For $p_0 = 1$ the denominator is equal to 0. For all $0 \leq p^+ \leq 1$ the limit is

$$
\lim_{p^+ \to 1} \rho(0, 0, 3) = 0.
$$

(18)

Second, we analyze the case $p_1 < p_0$. Thus, we set $p_1 = p^+ p_0$ for $0 \leq p^+ \leq 1$ yielding a correlation coefficient

$$
\rho(0, 0, 3) = \frac{(p_0 p^+ - 1) p_0 (p^+ - 1)}{p_0 p^+ - 1}.
$$

(19)

The correlation coefficient is $\rho(0, 0, 3) \leq 0$ for all $0 \leq p^+ \leq 1$.

**C. Interference Correlation for Case $(0, 1, 3)$**

We now assume that each node suffers in every slot under independent Rayleigh fading. Due to the independence of fading in consecutive slots, the overall variance of interference is increased and therefore the correlation coefficient is reduced. The variance of interference with independent Rayleigh fading is derived in [7], [32], and is given by

$$
E[\text{var} (I | \mathcal{N})] \equiv \kappa^2 \cdot \lambda \frac{\alpha \pi}{\alpha - 1} \cdot \frac{p_0(2 - 2p_1 + p_0)}{(1 - p_1 + p_0)^2}.
$$

(20)

Dividing (13) by this variance yields

$$
\rho(0, 1, 3) = \frac{(1 - p_1)(p_1 - p_0)}{2 - 2p_1 + p_0}.
$$

(21)

Similar to case $(0, 0, 3)$ we distinguish between reduced and increased retransmission probabilities. For an increased retransmission probability, we substitute $p_1 = (1 - p^+) p_0 + p^+$ into (21), which leads to

$$
\rho(0, 1, 3) = \frac{p^+ (1 - p^+) (1 - p_0)^2}{(1 - p^+) (2 - p_0) + p_0 p^+}.
$$

(22)

For a decreased retransmission probability, we substitute $p_1 = p^+ p_0$ into (21) and get

$$
\rho(0, 1, 3) = \frac{p_0 (p^+ - 1) (1 - p^+ p_0)}{2 - 2p_0 p^+ + p_0}.
$$

(23)

**D. Interference Correlation for Case $(0, 2, 3)$**

Last but not least, we have a constant channel over $c \geq 2$ slots. We assume that the channel changes of different nodes are not all in synchrony. Furthermore, we subdivide the set $S_{11}$ into the sets of nodes having the same channel states in both slots and nodes having different channel states. These sets are denoted by $S_{11}'$ and $S_{11}''$, respectively. Hence, we have $S_{11} = S_{11}' \cup S_{11}''$. The fractions of nodes out of $S_{11}$ that are within $S_{11}'$ and $S_{11}''$ are derived in [7], [46]. Combining this result with (5) yields

$$
S_{11}' = \frac{p_0 p_1 (1 + p_0 (c - 1) - p_1)}{(1 + p_0 - p_1) (1 + c p_0 - p_1)},
$$

(24)

$$
S_{11}'' = \frac{p_0^2 p_1}{(1 + p_0 - p_1) (1 + c p_0 - p_1)}.
$$

(25)
\( p_0 \) with \( p_1 > p_0 \).

Fig. 1: Interference correlation in case \((0, 0, 3)\).

\( p_0 \) with \( p_1 < p_0 \).

Fig. 2: Interference correlation in case \((0, 1, 3)\).

\( p_0 \) with \( p_1 > p_0 \).

Fig. 3: Interference correlation in case \((0, 2, 3)\).
By substituting these fractions into the derivation presented in Section III-B we calculate the correlation coefficient \( \rho(0, 2, 3) \). This correlation coefficient is

\[
\rho(0, 2, 3) = \frac{p_0 (1 + c p_0) + p_1^2 (4 + p_0(1 + 2c)) - 2p_1^2}{(2 - 2p_1 + p_0)(p_1 - c p_0 - 1)}
+ \frac{p_1 (p_0^2(1 - 2c) - 2p_0(1 + c) - 1)}{(2 - 2p_1 + p_0)(p_1 - c p_0 - 1)}.
\]

(26)

We again substitute \( p_1 = (1 - p^+)p_0 + p^+ \) for increased and \( p_1 = p^-p_0 \) for decreased retransmission probability into (26); the resulting equations are not presented.

IV. INTERPRETATION OF THE CORRELATION COEFFICIENT

In the following we present plots of the equations derived above and describe their behavior. In Figures 1 to 3 lines denote theoretical results while points indicate simulation results. As can be seen in the plots, theory generally matches well to simulations we performed to complement derivations.

Figure 1 plots the correlation coefficient for case \((0, 0, 3)\). The correlation coefficient linearly increases with \( p_1 \) and linearly decreases with \( p_0 \). If \( p_1 \geq p_0 \) (Figure 1(a)) the correlation depends on the increase of the transmission probability after a node has sent. If it is not changed (\( p^+ = 0 \)) there is no correlation. Otherwise, a positive correlation is introduced. If \( p_1 < p_0 \) (Figure 1(b)) the sending probability of a given node is decreased after a transmission. This results in a negative temporal correlation of interference. In the extreme case where a node always stays quiet directly after a transmission (\( p^- = p_1 = 0 \)), the correlation coefficient is \(-p_0\).

Case \((0, 1, 3)\) is similar to case \((0, 0, 3)\) except that independent Rayleigh block fading is introduced. As we know from previous work [6], [7], this kind of fading reduces the correlation by increasing the variance of interference. Figure 2 confirms this result (mind the different scale compared to Figure 1). For both reduced and increased retransmission probabilities we have similar trends to case \((0, 0, 3)\). However, the correlation shows a non-linear dependence on the transmission probabilities, which stems from the non-linear influence of these probabilities on the variance of interference.

In contrast to these two cases, case \((0, 2, 3)\) introduces an additional cause for interference correlation: a slowly changing channel. In Figure 3 plots of the correlation are shown, for which the channel stays constant for \( c = 4 \) slots before changing. This case can be interpreted as a mixture of case \((0, 0, 3)\), in which the channel stays constant forever \((c \rightarrow \infty)\), and case \((0, 1, 3)\), in which the channel changes in each slot \((c = 1)\). The correlation again shows a non-linear dependence on the transmission probabilities due to fading.

V. PRACTICAL IMPLICATIONS

We extended recent research results on the temporal behavior of interference considering correlation of traffic. Here, the transmission probability of a given node is altered if it has sent in the preceding slot. This change could be introduced, e.g., by a retransmission protocol that sends a packet again when lost. If the protocol is built in a way that a positive correlation is caused, this could diminish its efficiency: While constructed to counteract bad channel conditions, it introduces a correlation that again leads to similar conditions at the second transmission. If, however, the protocol is designed in a way to achieve a negative correlation, this might increase its performance significantly. It is therefore important to consider the dynamics of interference when constructing such protocols.

The intention of this paper is to provide the means to analyze protocols with regard to their influence on interference dynamics in a network. Further investigations could apply these methods to analyze existing protocols.

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